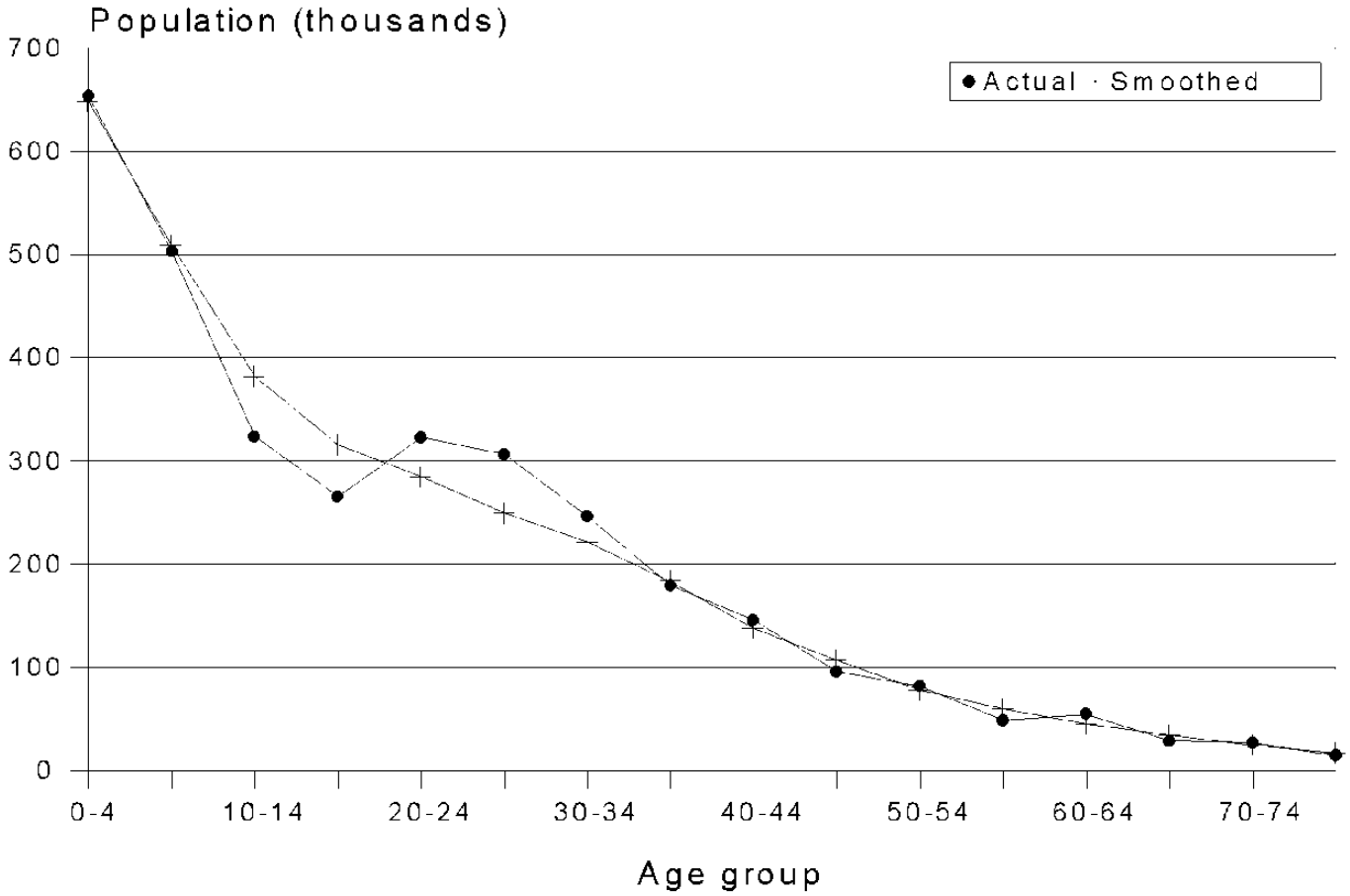
Demographic Analysis

Age and Sex Structure: Smoothing Techniques to Correct for Age Misreporting

Figure II-9. Actual and Smoothed Population with Strong Smoothing



Age Misreporting and Smoothing - Introduction

In this lesson, we consider techniques for smoothing the population age distribution when there are errors in age reporting.

- What do we hope to accomplish by smoothing?
- Reasons to smooth, and not to smooth
- Smoothing adjusts for *misreporting*, not *underreporting*
- Types of smoothing methods:
  - Light vs. strong
  - Preserve vs. modify slightly the population size
- Tips for deciding whether smoothing is needed and which method might be most appropriate

Why might we want to smooth?

Reasons to smooth:

- When ages are genuinely misreported
  - Planning and policies that require accurate counts by age may be adversely affected. Examples:
    - Children entering school system
    - Young males reaching military draft age
    - Qualification for older age public benefits
  - Flawed age-sex structures, when projected into the future, will also be flawed
  - Flawed age counts used as denominators in demographic rates (e.g. child mortality) may bias those rates

Why might we not want to smooth?

Reasons *not* to smooth:

- Sometimes odd-looking fluctuations in population age distributions **are real** – not due to misreporting.
- The most common features of age misreporting – digit preference – can be addressed by aggregating the population across 5-year or 10-year ages groups.
- Age misreporting might be confined to a particular age group.
- Smoothing assumes age misreporting, not under-reporting. If there is age/sex-specific **under-reporting**, smoothing inappropriately spreads it out.

Methods for Correcting Age Misreporting

Once the decision has been made to use smoothing to correct for age misreporting, there are several varieties available to do so.

Most smoothing techniques involve the application of a formula or function to the original data.

Misreporting involves a shift from one's actual age to another age, typically an adjacent age group. Formulas correct for that bias by using weighted averages based on counts at each age group as well as adjacent age groups.

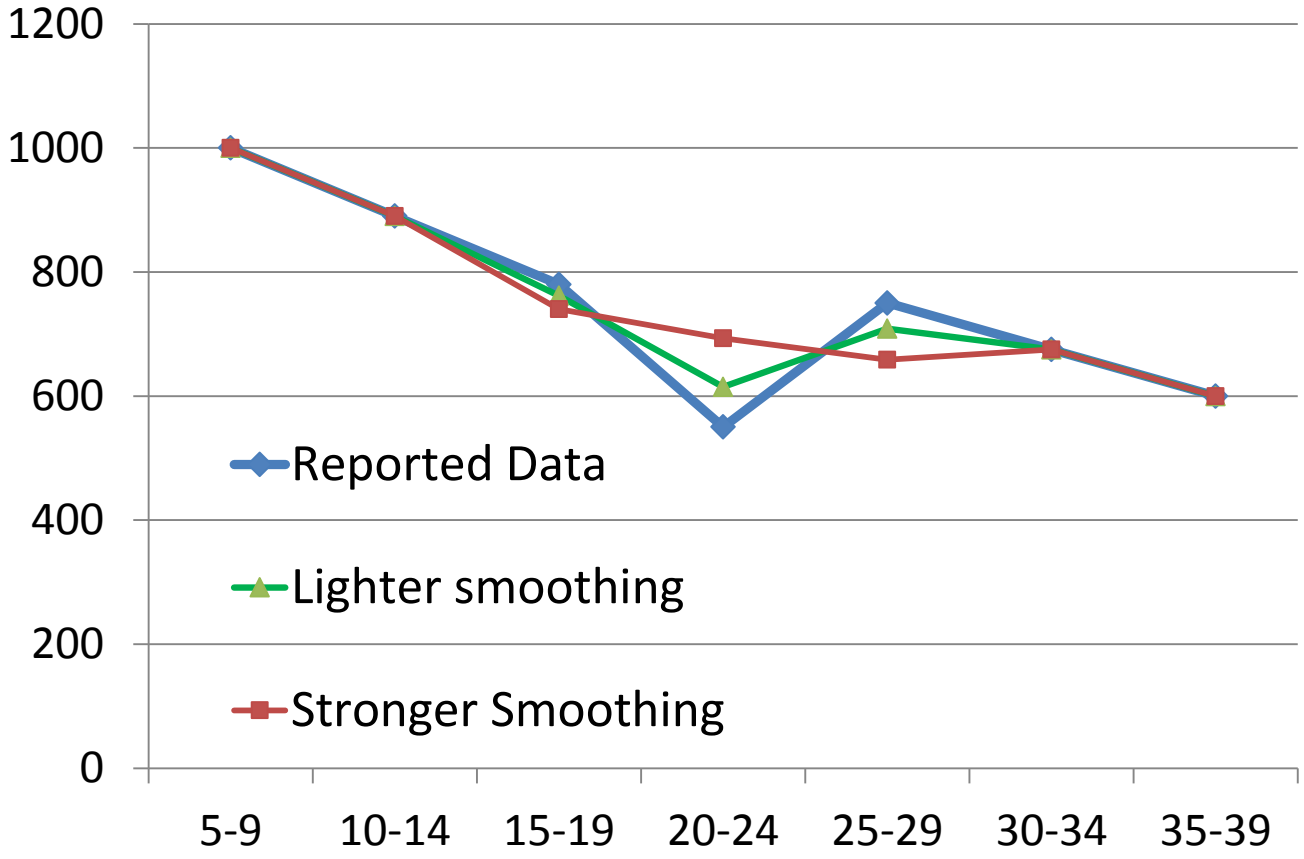
Methods for Correcting Age Misreporting - Types

Light or Strong?

Smoothing techniques may be lighter or stronger depending on the formulas that construct the weighted averages:

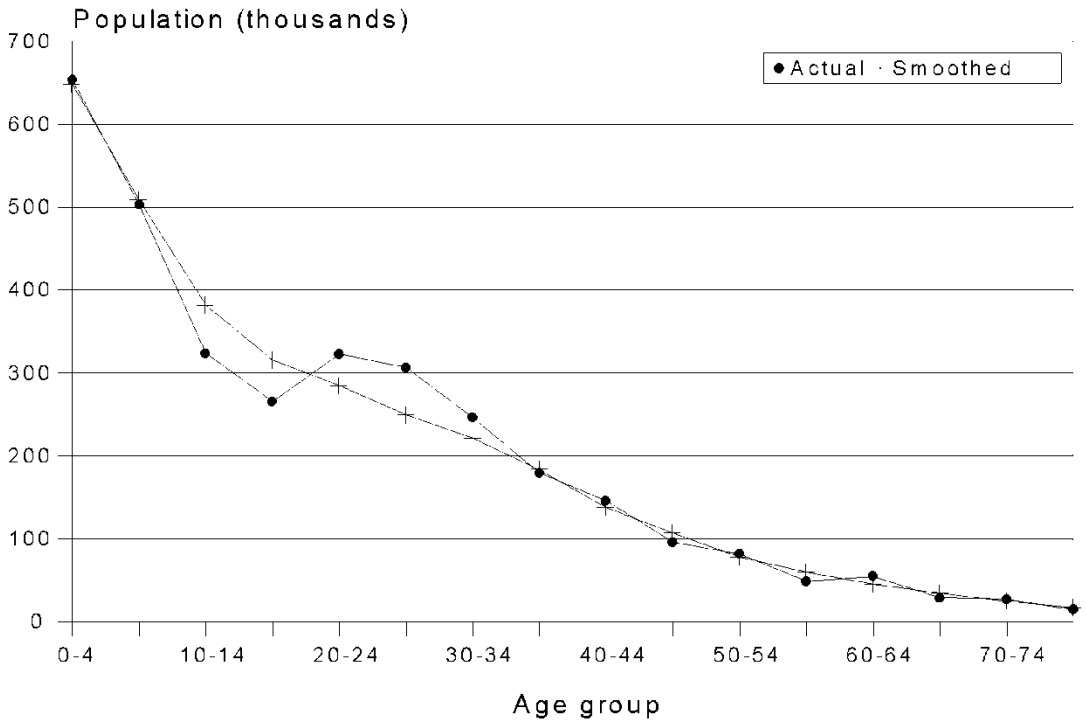
- Light Smoothing – formulas that give the greatest weight to what was reported for the age group in question and smallest weight to adjacent age groups
- Strong Smoothing – formulas that give greater weight to adjacent age counts and/or over wider age intervals. Resulting pattern does not follow the contours of reported data as well as lighter smoothing.

Example of Lighter vs. Stronger Smoothing of Age Data



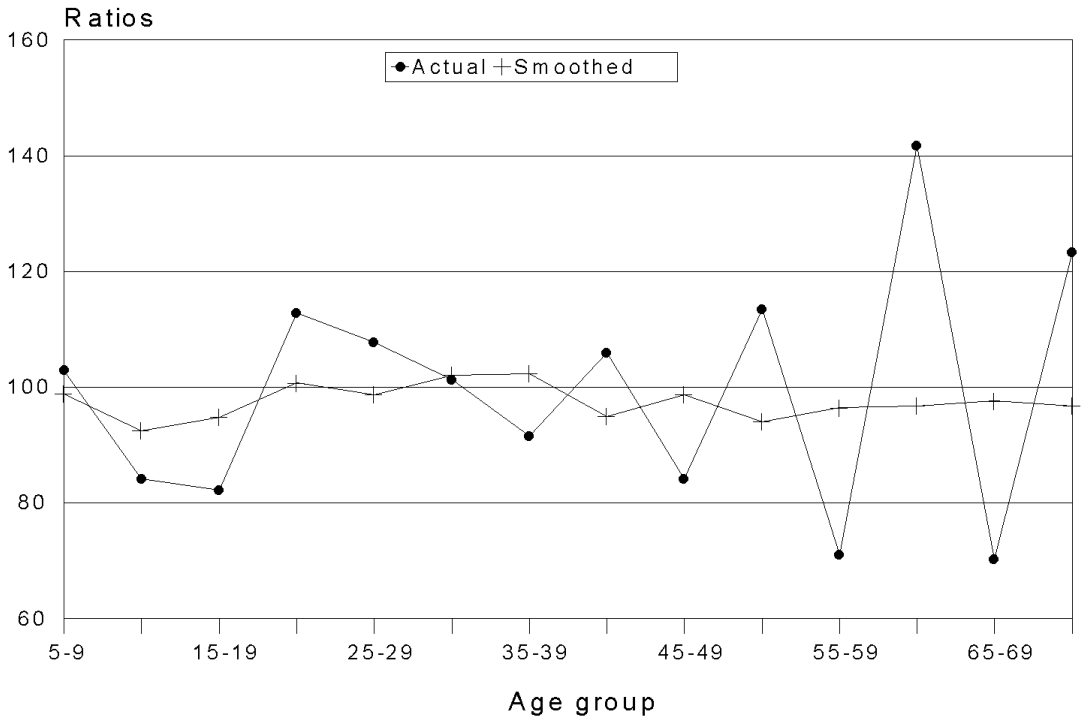
Arriaga's Strong Smoothing Formula

Figure II-9. Actual and Smoothed Population with Strong Smoothing



Smoothing Algorithms Compared

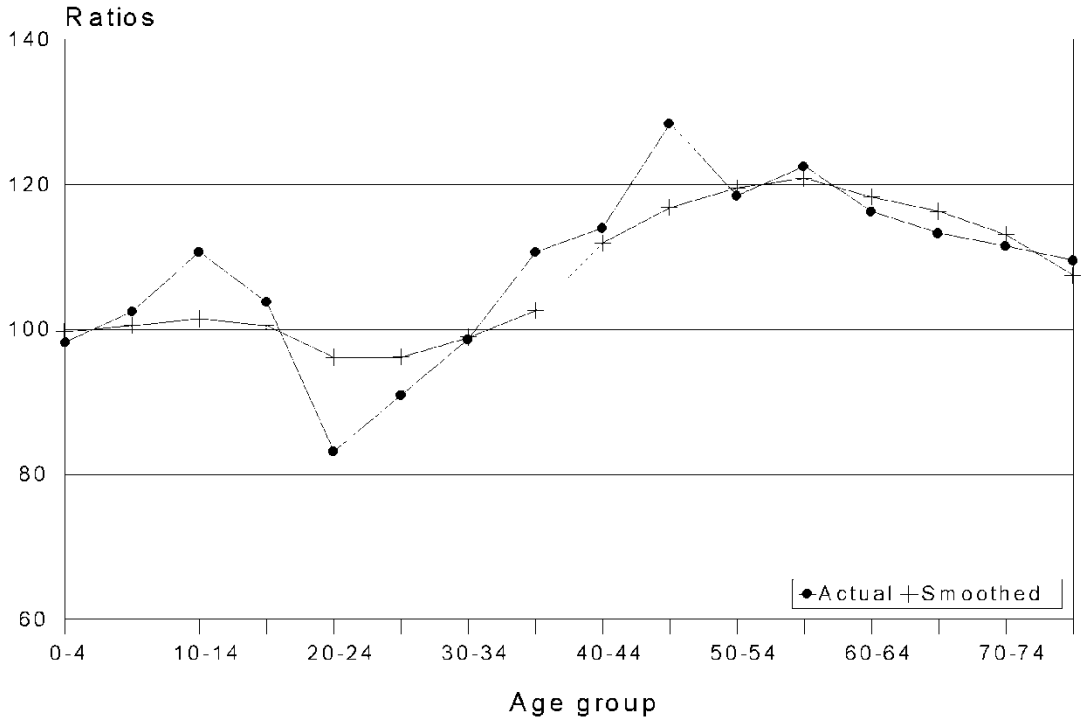
Figure II-13. Age Ratios: Actual Population and with Strong Smoothing



Note: Each ratio represents the population in a given age group divided by the average population of the two adjacent age groups, times 100.

Smoothing Algorithms Compared

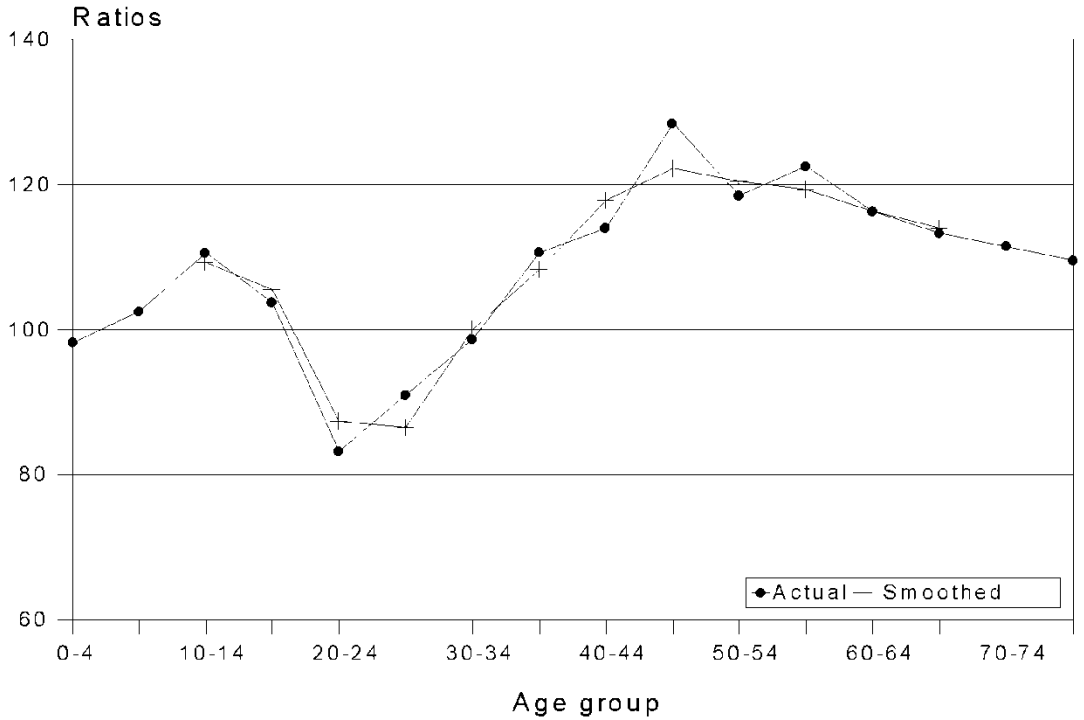
Figure II-11. Sex Ratios: Actual Population and with Strong Smoothing



Note: Each ratio refers to the number of males per 100 females in each age group.

Smoothing Algorithms Compared

Figure II-10. Sex Ratios: Actual Population and with Light Smoothing



Note: Each ratio refers to the number of males per 100

Methods for Correcting Age Misreporting - Types

Modify or Preserve Population Totals?

Most smoothing techniques can also be classified into two main categories:

- Techniques which accept the population in each 10-year age group and separate it into two 5-year age groups without modifying the total population size; and
- Techniques which smooth the 5-year age groups and, in the process, modify slightly (increasing or decreasing) the population being smoothed.

Methods that Do or Do Not Preserve Population Totals

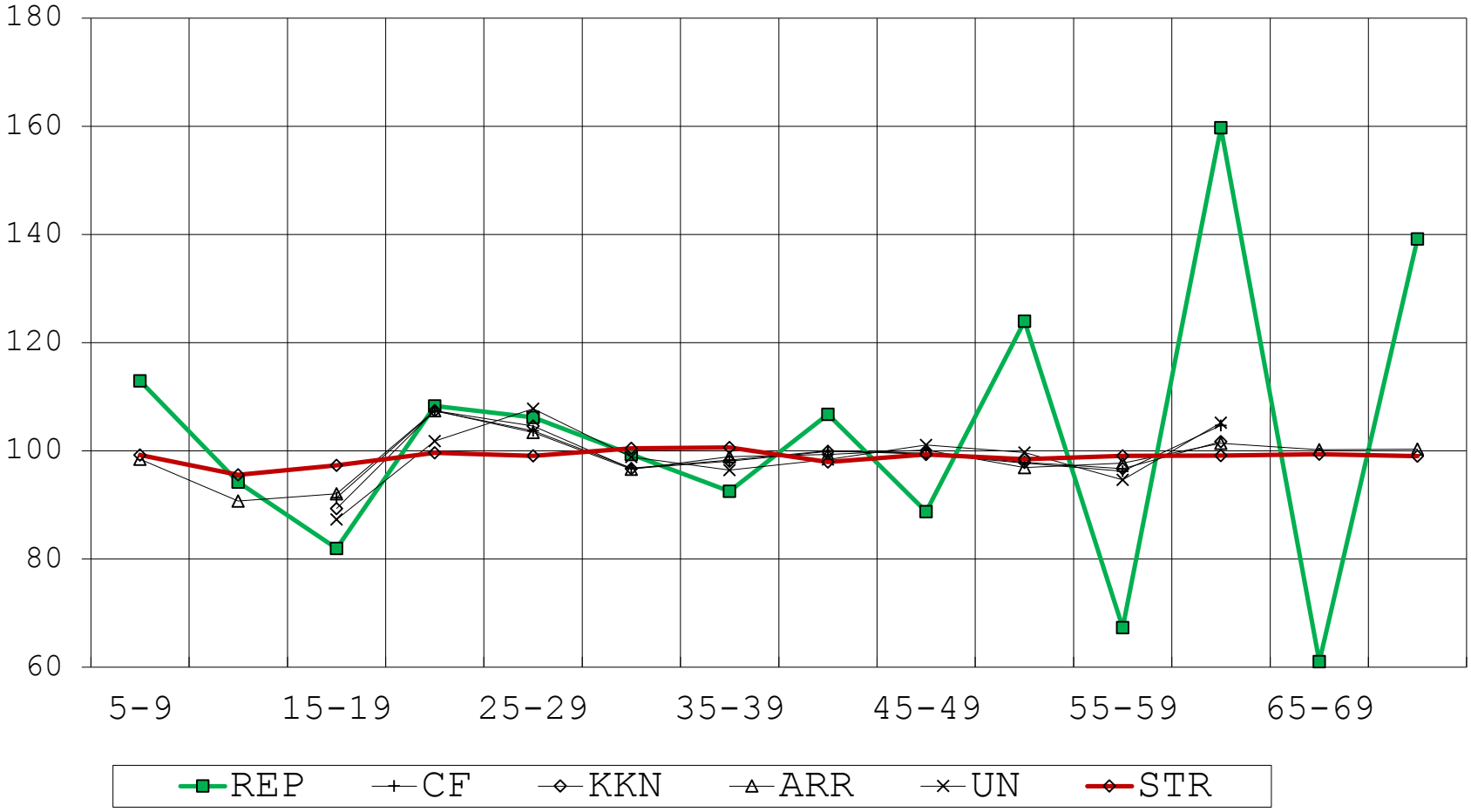
Methods that preserve the original total:

- The Carrier-Farrag and Karup-King-Newton (Carrier and Farrag, 1959) formulas do not separate the first or last 10-year age groups.
- The Arriaga (1968) formula.
- Arriaga's "strong smoothing" formula doesn't, but the results can be adjusted to agree with the total population over the range that is smoothed.

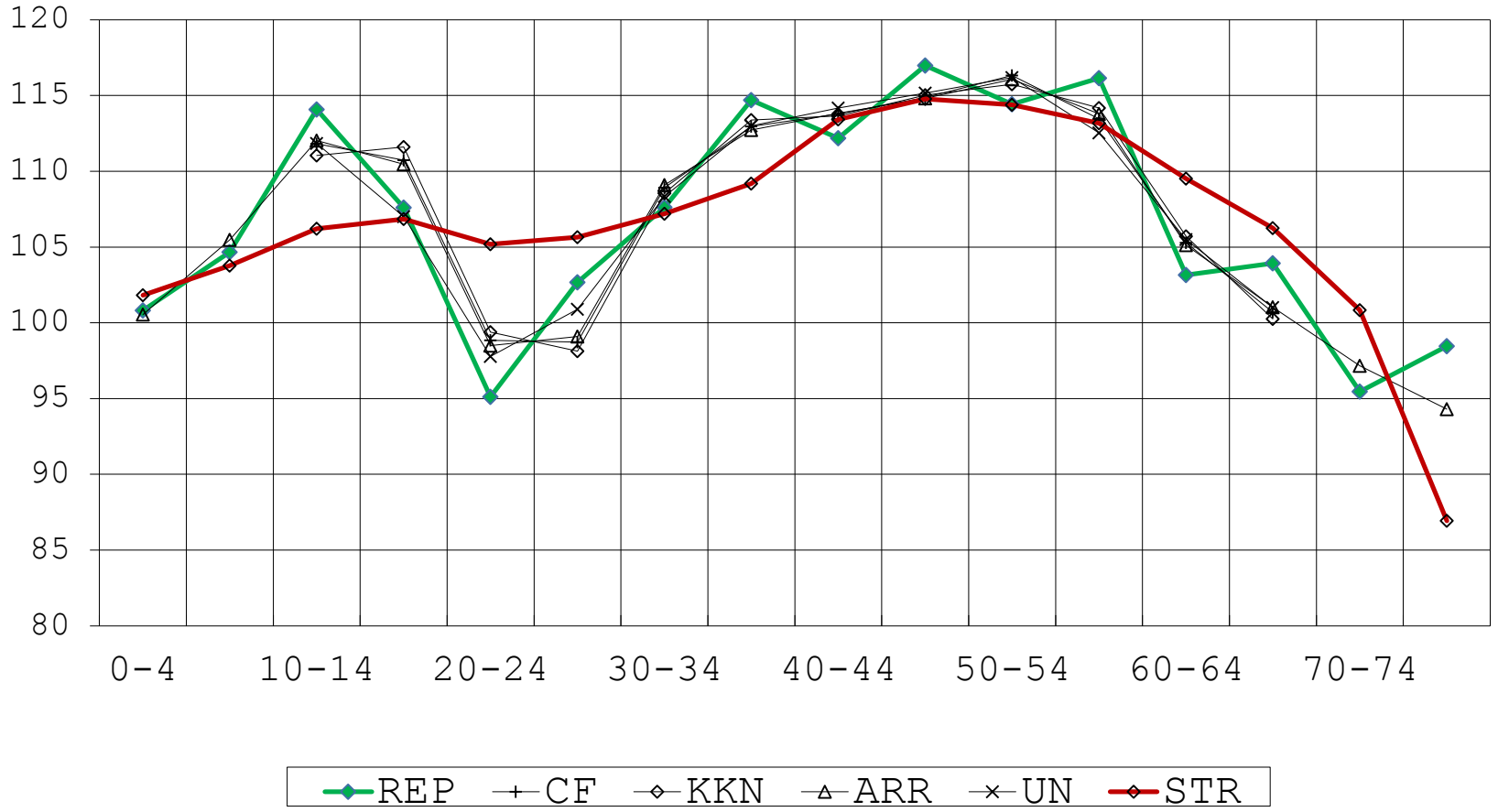
Methods that alter the total slightly:

- The United Nations method (1956, p. 12)

India - Female Age Ratios by Age: 1961 Census
REPORTED vs. SMOOTHED



India - Sex Ratios by Age: 1961 Census
REPORTED vs. SMOOTHED (based on Male % Female Pops.)



The Carrier-Farrag Formula

The Carrier-Farrag technique is based on the assumption that the relationship of a 5-year age group to its constituent 10-year age group is an average of similar relationships in three consecutive 10-year age groups.

$${}_5P_{x+5} = {}_{10}P_x / [1 + ({}_{10}P_{x-10} / {}_{10}P_{x+10})^{1/4}]$$ and

$${}_5P_x = {}_{10}P_x - {}_5P_{x+5}$$

where:

${}_5P_{x+5}$ represents the population at ages x+5 to x+9;

${}_{10}P_x$ represents the population at ages x to x+9; and

${}_5P_x$ represents the population at ages x to x+4.

The Karup-King-Newton Formula

The Karup-King-Newton formula assumes a quadratic relationship among each three consecutive 10-year age groups.

$$ {}_5P_x = \frac{1}{2}\,{}_{10}P_x + \frac{1}{16}\left({}_{10}P_{x-10} - {}_{10}P_{x+10}\right) \text{ and} $$

$$ {}_5P_{x+5} = {}_{10}P_x - {}_5P_x $$

Where ${}_5P_x$ is the first of two 5-year age groups comprising a 10-year age group ${}_{10}P_x$.

Arriaga's Light Smoothing Formula

Arriaga's formula assumes that a second degree polynomial passes by the midpoint of each three consecutive 10-year age groups and then integrates a 5-year age group.

In the PAS workbook AGESMTH the 10-year age groups start with age group 10-19. However, if there is strong age heaping on ages ending in 0, then the excess 10-year olds will be distributed over the ages 10-19, while in fact some should be distributed to ages below 10.

The new workbook ARRSM5 does this by smoothing starting with the 10-year age group 5-14.

Arriaga's Light Smoothing Formula

When the 10-year age group to be separated is the central group of three, the following formulas (Arriaga, 1968) are used:

$${}_5P_{x+5} = (-{}_{10}P_{x-10} + 11\ {}_{10}P_x + 2\ {}_{10}P_{x+10}) / 24 \quad \text{and}$$

$${}_5P_x = {}_{10}P_x - {}_5P_{x+5}$$

where:

${}_5P_{x+5}$ is the population ages x+5 to x+9;

${}_{10}P_x$ is the population ages x to x+9; and

${}_5P_x$ represents the population at ages x to x+4.

Arriaga's Light Smoothing Formula

When the 10-year age group to be separated is an extreme age group (the youngest or the oldest), the formulas are different. For the youngest age group, the following formulas are used:

$${}_5P_{x+5} = (8\ {}_{10}P_x + 5\ {}_{10}P_{x+10} - {}_{10}P_{x+20}) / 24 \quad \text{and}$$

$${}_5P_x = {}_{10}P_x - {}_5P_{x+5}$$

For the oldest age group, the coefficients are reversed:

$${}_5P_x = (-\ {}_{10}P_{x-20} + 5\ {}_{10}P_{x-10} + 8\ {}_{10}P_x) / 24 \quad \text{and}$$

$${}_5P_{x+5} = {}_{10}P_x - {}_5P_x$$

The United Nations Formula

United Nations (1956, p. 12) developed the following formula:

$$ {}_5P'_x = (- {}_5P_{x-10} + 4\ {}_5P_{x-5} + 10\ {}_5P_x + 4\ {}_5P_{x+5} - {}_5P_{x+10}\ ) /16 $$

where:

${}_5P'_x$ represents the smoothed population ages x to x+4.

Arriaga's Strong Smoothing Formula

If a more aggressive smoothing is desired (Arriaga, 1968), this can be achieved with the following formula:

$$ {}_{10}P'_x = ({}_{10}P_{x-10} + 2\ {}_{10}P_x + {}_{10}P_{x+10}) / 4 $$

Where:

${}_{10}P'_x$ represents the smoothed population ages x to x+9.

Arriaga's Strong Smoothing Formula

After getting the new smoothed 10-year age groups, the results can be adjusted to add up to the total for ages 10-69. These adjusted populations for 10-year age groups can then be split into 5-year ages using one of several formulas. AGESMTH uses the Arriaga quadratic formula.

Smoothing Single Years of Age

- In cases where there is very strong age-heaping on certain ages (e.g., 0's and 1's), it may make sense to try to smooth the population by single years of age.
- Alternatively, the smoothed 5-year ages can be split into single years of age using various methods.

Smoothing Single Years of Age

- The NewPAS workbook POP1SM.xls uses a weighted moving average to get a smoothed population by single years of age.
- The workbook allows the user to input the desired weights. Different weights will result in more or less smoothing.

Smoothing Single Years of Age

The formula used is:

$$S(x) = \frac{\sum_{i=-5}^{+5} w(i) \times P(x+i)}{\sum_{i=-5}^{+5} w(i)}$$

The formula is modified to use "trimmed weights" that involve fewer age groups at the beginning and end of the age distribution.

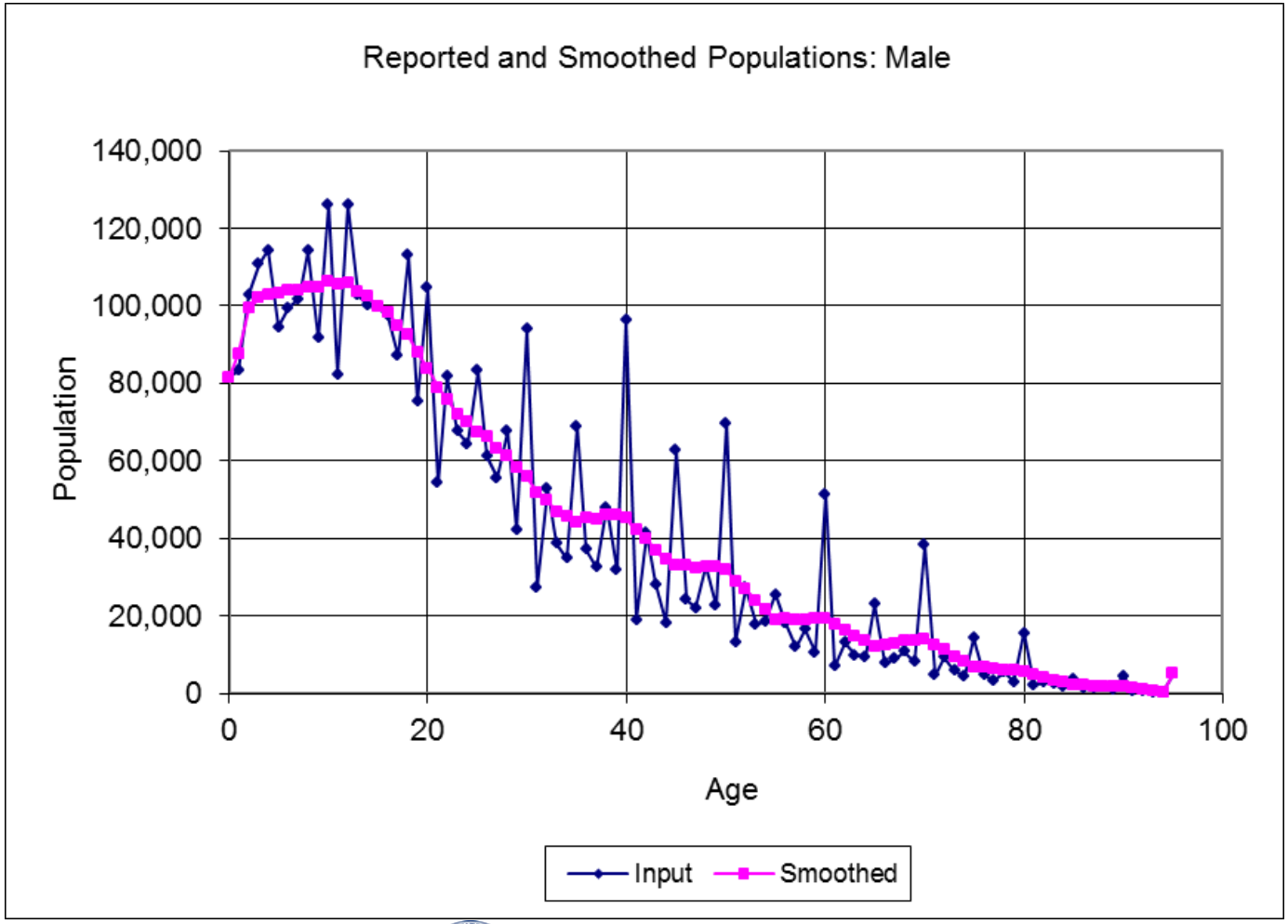
Smoothing Single Years of Age

- The default weights, w(i) are:

Relative age	Weight	Trimmed weights			
Sum	100.00	100.00	100.00	100.00	100.00
-5	0.00				
-4	4.00	4.00			
-3	8.00	8.00	6.25		
-2	12.00	12.00	12.50	11.11	
-1	16.00	16.00	18.75	22.22	25.00
0	20.00	20.00	25.00	33.33	50.00
1	16.00	16.00	18.75	22.22	25.00
2	12.00	12.00	12.50	11.11	
3	8.00	8.00	6.25		
4	4.00	4.00			
5	0.00				

Smoothing Single Years of Age

Default Weights for Haiti: 2003



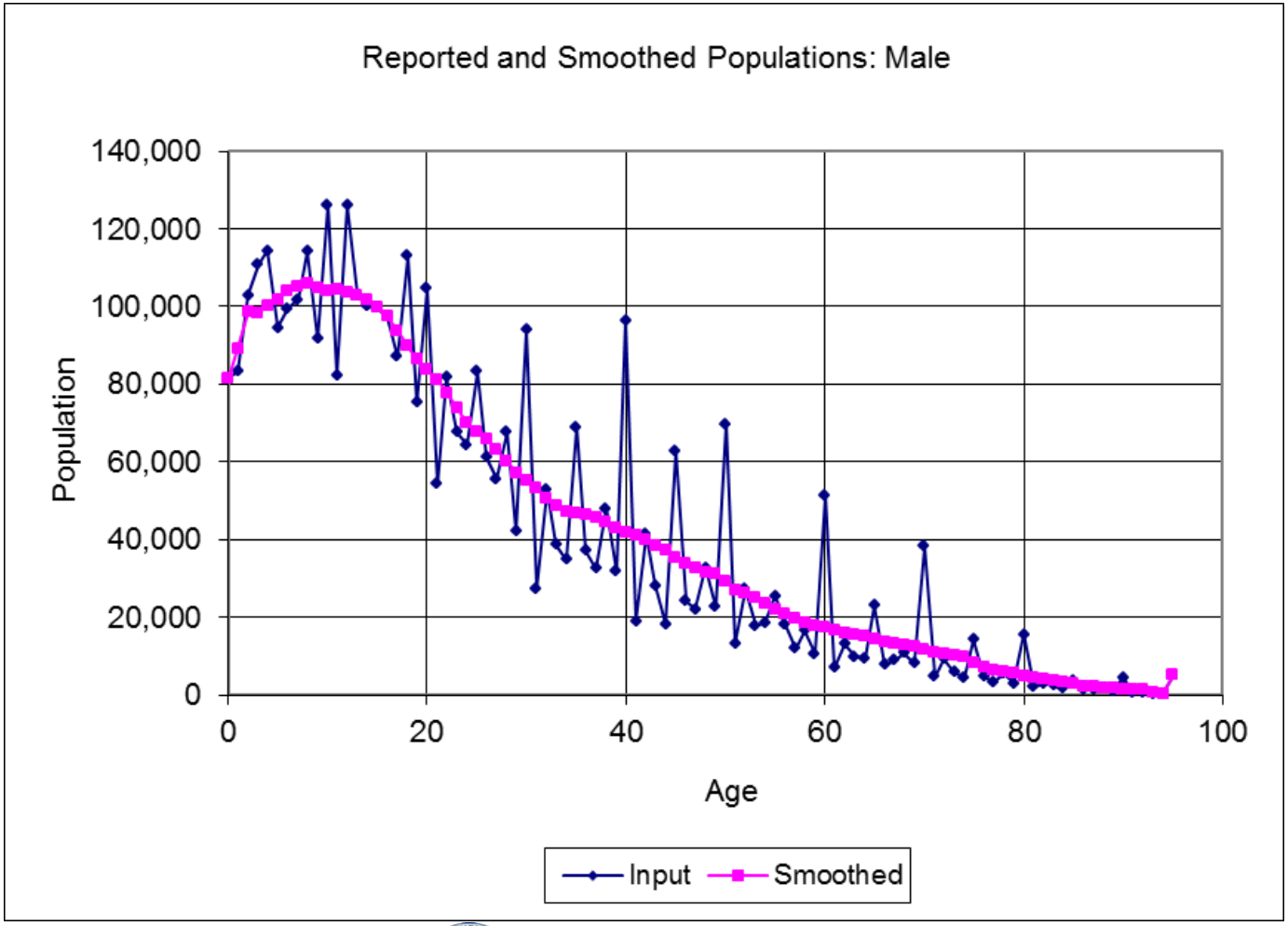
Smoothing Single Years of Age

- Alternative weights for Haiti 2003

Relative age	Weight	Trimmed weights			
Sum	90.00	90.00	90.00	90.00	90.00
-5	4.50				
-4	9.00	10.00			
-3	9.00	10.00	12.86		
-2	9.00	10.00	12.86	18.00	
-1	9.00	10.00	12.86	18.00	30.00
0	9.00	10.00	12.86	18.00	30.00
1	9.00	10.00	12.86	18.00	30.00
2	9.00	10.00	12.86	18.00	
3	9.00	10.00	12.86		
4	9.00	10.00			
5	4.50				

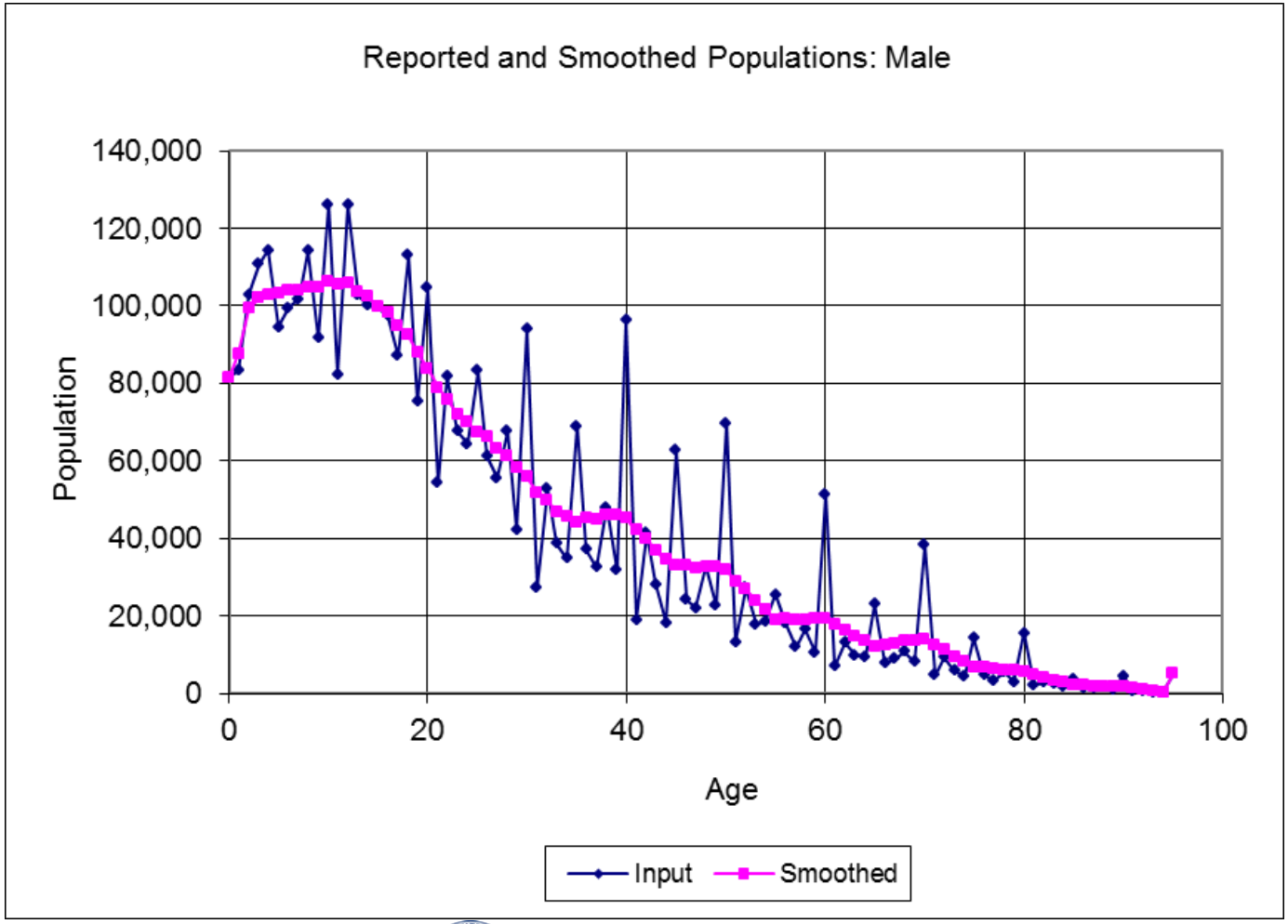
Smoothing Single Years of Age

Alternative Weights for Haiti: 2003



Smoothing Single Years of Age

Default Weights for Haiti: 2003



To Smooth or Not to Smooth, and if So, How? - Tips

- The decision must be made on a case by case basis given the context at hand.
- Step 1 - Look at your data. Graph the age and sex distributions using programs discussed earlier (PYRAMID, Pyr2, GRPOP-YB, AGESEX).
- In general, a regular saw-tooth pattern across successive age groups provide a good rationale for smoothing.
- Comparisons across successive censuses and a knowledge of past trends of mortality, fertility, and migration will help in appraising the accuracy of the reported age and sex structure of the population.

To Smooth or Not to Smooth? - Cautions

- Caution – Since strong smoothing may erase actual demographic history, a decision to use it should be considered very carefully.
- Caution – Even if smoothing produces more plausible age distributions, it may not improve distortions in sex ratios by age (and vice versa).
- Caution - The population age distribution may not need to be fully smoothed across all age groups if only part of it is considered problematic.
- Caution – If underreporting exists at a particular age, instead of smoothing one may need “filling.”

Population Smoothing Software

- PAS
  - AGESMTH.xls
- NewPAS
  - ARRSM5.xls
  - POP1SM.xls

Exercises

- Apply the various smoothing methods to the results of two censuses from your country (if possible).
- Decide which method (if any) seems to work best for your data.
- Does your data need smoothing at all ages?