# Demographic Analysis

## Mortality:

## The Life Table

## Its Construction and Applications

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# Mortality: Introduction

What is a life table?

A table that displays the life expectancy and the probability of dying at each age (or age group) for a given population, according to the age-specific death rates prevailing at that time. The life table gives an organized, complete picture of a population's mortality.

Source: Population Reference Bureau, Glossary of Demographic Terms, http://www.prb.org/Publications/Lesson-Plans/Glossary.aspx

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# Mortality: Introduction

In this session, we will examine the key components of the **life table** – a critical tool in demographic analysis

- Age-specific deaths rates (from deaths and population)
- Conversion of rates into probabilities or age-specific mortality rates based on “separation factors”
- Implied survival proportions from a hypothetical birth cohort (100,000)
- Total years lived by that cohort at each age
- Life expectancy at birth (and at other ages)

# Life Table Construction: From $_nM_x$ to $e_x$

| x | n | $_nM_x$ | $_nk_x$ | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $_5P_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0.1072 | 0.33 | 0.1000 | 100000 | 10000 | 93300 | 0.901 | 5630649 | 56.3 |
| 1 | 4 | 0.0034 | 1.56 | 0.0135 | 90000 | 1219 | 357023 | 0.983 | 5537349 | 61.5 |
| 5 | 5 | 0.0010 | 2.50 | 0.0049 | 88781 | 438 | 442809 | 0.996 | 5180326 | 58.3 |
| 10 | 5 | 0.0007 | 2.50 | 0.0037 | 88343 | 324 | 440903 | 0.994 | 4737517 | 53.6 |
| 15 | 5 | 0.0017 | 2.50 | 0.0084 | 88019 | 741 | 438242 | 0.988 | 4296614 | 48.8 |
| 20 | 5 | 0.0030 | 2.50 | 0.0147 | 87278 | 1283 | 433182 | 0.984 | 3858372 | 44.2 |
| 25 | 5 | 0.0036 | 2.50 | 0.0181 | 85995 | 1553 | 426092 | 0.978 | 3425190 | 39.8 |
| 30 | 5 | 0.0054 | 2.50 | 0.0268 | 84442 | 2266 | 416545 | 0.973 | 2999097 | 35.5 |
| 35 | 5 | 0.0054 | 2.50 | 0.0266 | 82176 | 2187 | 405411 | 0.952 | 2582553 | 31.4 |
| 40 | 5 | 0.0146 | 2.50 | 0.0705 | 79989 | 5635 | 385856 | 0.934 | 2177141 | 27.2 |
| 45 | 5 | 0.0128 | 2.50 | 0.0619 | 74354 | 4601 | 360265 | 0.907 | 1791286 | 24.1 |
| 50 | 5 | 0.0269 | 2.50 | 0.1262 | 69752 | 8802 | 326756 | 0.895 | 1431020 | 20.5 |
| 55 | 5 | 0.0170 | 2.50 | 0.0817 | 60950 | 4978 | 292305 | 0.864 | 1104264 | 18.1 |
| 60 | 5 | 0.0433 | 2.50 | 0.1954 | 55972 | 10935 | 252521 | 0.816 | 811959 | 14.5 |
| 65 | 5 | 0.0371 | 2.50 | 0.1699 | 45036 | 7651 | 206056 | 0.758 | 559439 | 12.4 |
| 70 | 5 | 0.0785 | 2.50 | 0.3281 | 37386 | 12266 | 156264 | 0.652 | 353383 | 9.5 |
| 75 | 5 | 0.0931 | 2.50 | 0.3775 | 25120 | 9483 | 101893 | 0.569 | 197119 | 7.8 |

# Life Expectancy

- Expectation of further life beyond age x
- The average number of additional years a person could expect to live if current mortality trends were to continue for the rest of that person's life.
- Most common: life expectancy at birth or $e_0$
- Why would this be a useful measure?

# 1. Age-specific Death Rates

We start the calculation of the life table with age-specific death rates, ${}_nM_x$

Age-specific central death rates are calculated as the number of deaths in a particular age group – typically during a year – per 1,000 population in that same age group (best measured at midyear):

$${}_nM_x = {}_nD_x / {}_nP_x$$

# What is the $_5M_{10}$?

| Age Group | Male Deaths | Male Population |
|---|---|---|
| 0 | 57,200 | 1,260,897 |
| 1-4 | 18,526 | 6,005,011 |
| 5-9 | 9,005 | 8,730,750 |
| 10-14 | 6,400 | 8,284,907 |
| 15-19 | 8,358 | 6,603,607 |
| 20-24 | 6,249 | 5,461,651 |
| | | |

$_5M_{10}$=0.772 per 1000

Source: 2011 Bangladesh Census of Population and Housing

# 2. Probability of Dying – ${}_nq_x$

- Probability of dying within an interval of length n that starts at age x and ends at age x + n

# 2. Probability of Dying – $_nq_x$

A probability requires calculation of an appropriate population "at risk" for a certain event (in this case, a death).

To calculate probability of dying within a certain age interval, those at "at risk" of dying include:

1. Those who survived the interval (e.g. those counted in a census at the next age interval)
2. Those who died during the interval

Such probabilities can be calculated from age-specific death rates based on "separation factors" …

# 2. Probability of Dying – ${}_nq_x$

Probability of dying between exact ages: ${}_nq_x$

In symbols:

$$ {}_nq_x = \frac{n \quad {}_nM_x}{1 + (n - {}_nk_x) \quad {}_nM_x} $$

Where:

${}_nM_x$ is the age-specific death rate for ages x to x+n; and

${}_nk_x$ is the separation factor of deaths for ages x to x+n.

# 2. Probability of Dying – $_nq_x$

Probability of dying between exact ages: $_nq_x$

In symbols:

$$_nq_x = \frac{n \quad _nM_x}{1 + (n - {_nk_x}) \quad _nM_x}$$

Where:

$_nM_x$ is the age-specific death rate for ages x to x+n; and

$_nk_x$ is the separation factor of deaths for ages x to x+n.

# 3. Separation Factors ${}_nk_x$

The **"separation factor"** is the average number of years lived during an age interval by persons who die within that age interval.

- For most age groups, set to n/2 (or 2.5 if using 5-year age groups)
- Different for youngest (ages 0, 1-4) and oldest ages (open ended age group)

# Separation Factors at Youngest and Oldest Ages

However, at very young and very old ages, the likelihood of death within a given interval is not the same.

For instance, during infancy, the risk of death is especially high during the first month. Thus, for example, from birth to age 1, a separation factor of 0.33 would indicate that, on average, infants who died had lived for 1/3 of a year (or four months).

Problem with the open ended age interval

- Can't take n/2.
- Generally use $1/_{\infty}Mx$

# Now let's calculate $_nq_x$

| Age | n | nMx | nax | nqx |
|---|---|---|---|---|
| 10 | 5 | 0.00077 | 2.500 | 0.00386 |
| 15 | 5 | 0.00127 | 2.500 | |
| 20 | 5 | 0.00114 | 2.500 | 0.00570 |
| 25 | 5 | 0.00116 | 2.500 | 0.00578 |
| 30 | 5 | 0.00135 | 2.500 | |
| 35 | 5 | 0.00187 | 2.500 | 0.00931 |

5q15 =
0.00631

5q30 =
0.00673

# Difference between nMx and nqx

- Time interval
  - At most ages, ${}_nq_x$ will be *about* n times that of ${}_nM_x$.
  - ${}_nq_x$ are measured across an n-year age span
  - ${}_nM_x$ indicate average rate of death at each year in the span
- Denominator
  - ${}_nM_x$: the denominator of the death rate is based on mid-year counts of *survivors*,
  - ${}_nq_x$: the denominator is based on those *starting* an age interval

# Infant Mortality Rate (IMR) – a special case

For infants, those "at risk" of dying consist of births. When vital registration data are available, the infant mortality rate is thus often calculated as the ratio of the number of deaths of infants under 1 year of age ($D_0$) to the number of live births occurring that year (B), times 1,000,

$D_0$ / B * 1,000.

```
For example, the IMR for Chile in 1986 is
obtained as follows:

 (5,220 / 272,997) x 1,000 = 19.12
(infant
deaths)    (births)

There were 19 infant deaths per 1,000
live births in Chile in 1986.
```

# Life Table Construction: $_nq_x$

| x | n | $_nM_x$ | $_nk_x$ | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $_5P_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0.1072 | 0.33 | 0.1000 | 100000 | 10000 | 93300 | 0.901 | 5630649 | 56.3 |
| 1 | 4 | 0.0034 | 1.56 | 0.0135 | 90000 | 1219 | 357023 | 0.983 | 5537349 | 61.5 |
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| 25 | 5 | 0.0036 | 2.50 | 0.0181 | 85995 | 1553 | 426092 | 0.978 | 3425190 | 39.8 |
| 30 | 5 | 0.0054 | 2.50 | 0.0268 | 84442 | 2266 | 416545 | 0.973 | 2999097 | 35.5 |
| 35 | 5 | 0.0054 | 2.50 | 0.0266 | 82176 | 2187 | 405411 | 0.952 | 2582553 | 31.4 |
| 40 | 5 | 0.0146 | 2.50 | 0.0705 | 79989 | 5635 | 385856 | 0.934 | 2177141 | 27.2 |
| 45 | 5 | 0.0128 | 2.50 | 0.0619 | 74354 | 4601 | 360265 | 0.907 | 1791286 | 24.1 |
| 50 | 5 | 0.0269 | 2.50 | 0.1262 | 69752 | 8802 | 326756 | 0.895 | 1431020 | 20.5 |
| 55 | 5 | 0.0170 | 2.50 | 0.0817 | 60950 | 4978 | 292305 | 0.864 | 1104264 | 18.1 |
| 60 | 5 | 0.0433 | 2.50 | 0.1954 | 55972 | 10935 | 252521 | 0.816 | 811959 | 14.5 |
| 65 | 5 | 0.0371 | 2.50 | 0.1699 | 45036 | 7651 | 206056 | 0.758 | 559439 | 12.4 |
| 70 | 5 | 0.0785 | 2.50 | 0.3281 | 37386 | 12266 | 156264 | 0.652 | 353383 | 9.5 |
| 75 | 5 | 0.0931 | 2.50 | 0.3775 | 25120 | 9483 | 101893 | 0.569 | 197119 | 7.8 |

# 4. Survivors at exact age x: $l_x$

The next life table function to be calculated is the number of persons surviving to each exact age, or $l_x$

$l_0$ is called the radix and usually starts with 100,000 births (at exact age 0).

The number of survivors from birth to each exact age is obtained using the probabilities, ${}_nq_x$, as estimated above.

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# 4. Survivors at exact age x: $l_x$

Survivors at exact age x, $l_x$

In symbols:

$$l_{x+n} = l_x \, (1 - {}_nq_x)$$

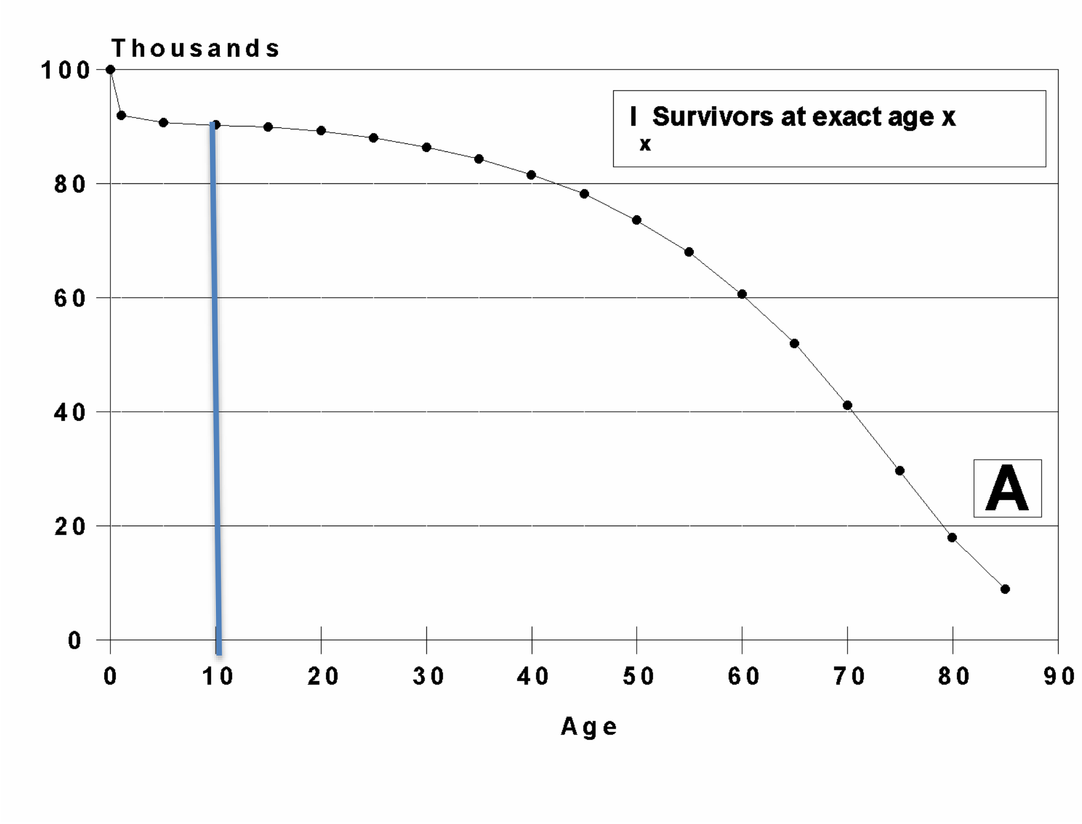
Where:

${}_nq_x$ is as defined above; and

The first $l_x$ is $l_0$ and usually is defined as 100,000.

# Life Table Construction: $l_x$

Figure III -3. Selected Life Table Functions



# Let's calculate lx

| x | n | nMx | nax | nqx | lx |
|---|---|---|---|---|---|
| - | - | - | - | - | - |
| 0 | 1 | 0.03471 | 0.139 | 0.03370 | 100,000 |
| 1 | 4 | 0.00309 | 1.757 | 0.01226 | |
| 5 | 5 | 0.00103 | 2.500 | 0.00514 | 95,446 |
| 10 | 5 | 0.00077 | 2.500 | 0.00386 | |
| 15 | 5 | 0.00127 | 2.500 | 0.00631 | 94,589 |
| 20 | 5 | 0.00114 | 2.500 | 0.00570 | 93,992 |
| 25 | 5 | 0.00116 | 2.500 | 0.00578 | 93,456 |
| 30 | 5 | 0.00135 | 2.500 | 0.00673 | 92,916 |

$l_1$ = 96,630

$l_{10}$ = 94,955

# 5. Life table deaths: ${}_nd_x$

Since $l_x$ represents the number of persons alive at each exact age x, the difference between two consecutive values (for example, $l_x$ and $l_{x+n}$) represents the number of deaths between the corresponding ages (x and x+n, in this case). This number of deaths between two exact ages is symbolized by ${}_nd_x$ in the life table.

# Life Table Construction: $_nd_x$

Deaths between two exact ages: $_nd_x$

In symbols:

$$_nd_x = l_x - l_{x+n}$$

Where:

$l_x$ is as defined above.

# Let's calculate ${}_n d_x$

| x | n | nMx | nax | nqx | lx | ndx | |
|---|---|---|---|---|---|---|---|
| - | - | - | - | - | - | - | |
| 0 | 1 | 0.03471 | 0.139 | 0.03370 | 100,000 | 3,370 | |
| 1 | 4 | 0.00309 | 1.757 | 0.01226 | 96,630 | 1,184 | |
| 5 | 5 | 0.00103 | 2.500 | 0.00514 | 95,446 | 491 | |
| 10 | 5 | 0.00077 | 2.500 | 0.00386 | 94,955 | | 366 |
| 15 | 5 | 0.00127 | 2.500 | 0.00631 | 94,589 | 597 | |
| 20 | 5 | 0.00114 | 2.500 | 0.00570 | 93,992 | 536 | |
| 25 | 5 | 0.00116 | 2.500 | 0.00578 | 93,456 | | 540 |
| 30 | 5 | 0.00135 | 2.500 | 0.00673 | 92,916 | 625 | |

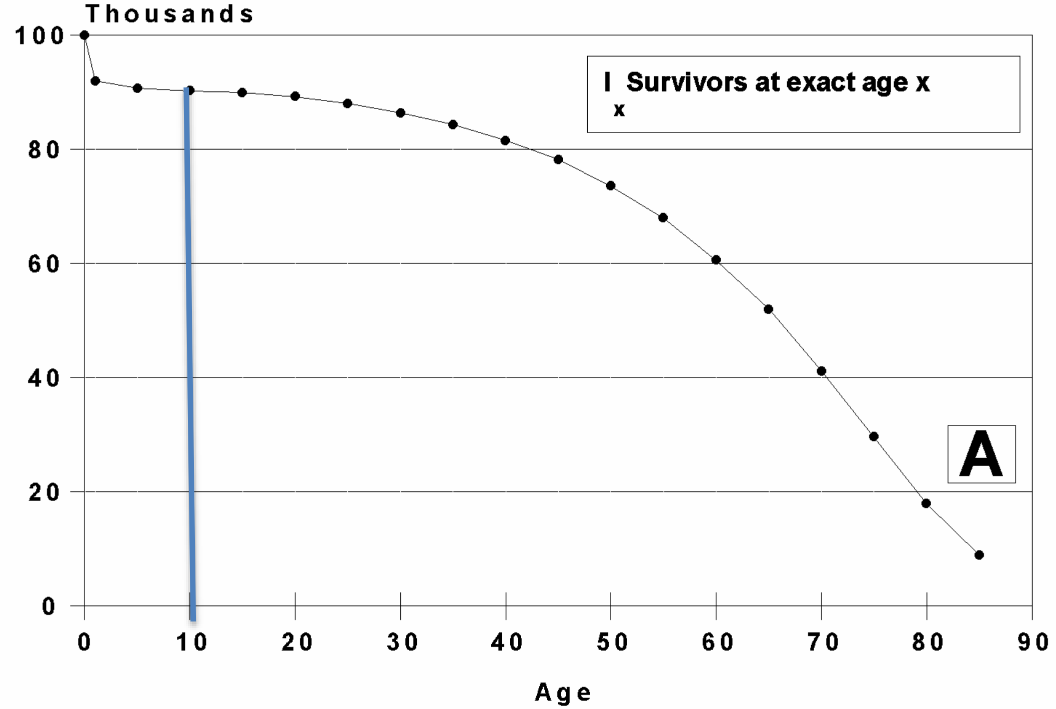
# 6. Person Years Lived: ${}_nL_x$

The total number of years that the single cohort of 100,000 would live between two specific ages, x to x+n.

Area under the lx curve

# Life Table Construction: $l_x$

Figure III -3. Selected Life Table Functions



# 6. Person Years Lived: ${}_{n}L_{x}$

The ${}_{n}L_{x}$ function can be computed as the sum of person-years lived by those that survive to age x+n and the person-years lived by those that die between those ages:

$$ {}_{n}L_{x} = n * l_{x+n} + {}_{n}k_{x} * {}_{n}d_{x} $$

# Let’s Calculate $_nL_x$

| x | n | nMx | nkx | nqx | lx | ndx | nLx | |
|---|---|---|---|---|---|---|---|---|
| - | - | - | - | - | - | - | - | |
| 0 | 1 | 0.03471 | 0.139 | 0.03370 | 100,000 | 3,370 | 97,100 | |
| 1 | 4 | 0.00309 | 1.757 | 0.01226 | 96,630 | 1,184 | 383,864 | |
| 5 | 5 | 0.00103 | 2.500 | 0.00514 | 95,446 | 491 | | 476,001 |
| 10 | 5 | 0.00077 | 2.500 | 0.00386 | 94,955 | 366 | 473,859 | |
| 15 | 5 | 0.00127 | 2.500 | 0.00631 | 94,589 | 597 | 471,452 | |
| 20 | 5 | 0.00114 | 2.500 | 0.00570 | 93,992 | 536 | 468,620 | |
| 25 | 5 | 0.00116 | 2.500 | 0.00578 | 93,456 | 540 | | 465,930 |
| 30 | 5 | 0.00135 | 2.500 | 0.00673 | 92,916 | 625 | 463,017 | |
| 35 | 5 | 0.00187 | 2.500 | 0.00931 | 92,291 | 859 | 459,307 | |

# 7. Person years of life remaining: $T_x$

The person years of life remaining may be calculated by summing all the $_nL_x$ values

Person years of life remaining for ages x and above: $T_x$

In symbols:

$$T_x = \sum_{i=x}^{w} {}_nL_i$$

Where:

$_nL_x$ is as defined above; and

w represents the oldest age.

# 8. Life Expectancy: $e_x$

- Expectation of further life beyond age x
- The average number of additional years a person could expect to live by those who are alive at each age x.
- The number of years that the life table population will live from age x up to the point when all have died, divided by the number of persons alive at exact age x,

# Life Table Construction: $e_x$

Life expectancy at age x: $e_x$

In symbols:

$$e_x = \frac{T_x}{l_x}$$

Where $T_x$ and $l_x$ are as defined above.

# Let's Calculate $e_x$

| x | n | nMx | nax | nqx | lx | ndx | nLx | Tx | ex |
|---|---|---|---|---|---|---|---|---|---|
| - | - | - | - | - | - | - | - | - | - |
| 0 | 1 | 0.03471 | 0.139 | 0.03370 | 100,000 | 3,370 | 97,100 | 7,558,096 | 75.58 |
| 1 | 4 | 0.00309 | 1.757 | 0.01226 | 96,630 | 1,184 | 383,864 | 7,460,996 | 77.21 |
| 5 | 5 | 0.00103 | 2.500 | 0.00514 | 95,446 | 491 | 476,001 | 7,077,132 | 74.15 |
| 10 | 5 | 0.00077 | 2.500 | 0.00386 | 94,955 | 366 | 473,859 | 6,601,130 | 69.52 |
| 15 | 5 | 0.00127 | 2.500 | 0.00631 | 94,589 | 597 | 471,452 | 6,127,272 | 64.78 |

# Life Table Construction: $e_x$

| x | n | $_nM_x$ | $_nk_x$ | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $_5P_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0.1072 | 0.33 | 0.1000 | 100000 | 10000 | 93300 | 0.901 | 5630649 | 56.3 |
| 1 | 4 | 0.0034 | 1.56 | 0.0135 | 90000 | 1219 | 357023 | 0.983 | 5537349 | 61.5 |
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| 30 | 5 | 0.0054 | 2.50 | 0.0268 | 84442 | 2266 | 416545 | 0.973 | 2999097 | 35.5 |
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| 65 | 5 | 0.0371 | 2.50 | 0.1699 | 45036 | 7651 | 206056 | 0.758 | 559439 | 12.4 |
| 70 | 5 | 0.0785 | 2.50 | 0.3281 | 37386 | 12266 | 156264 | 0.652 | 353383 | 9.5 |
| 75 | 5 | 0.0931 | 2.50 | 0.3775 | 25120 | 9483 | 101893 | 0.569 | 197119 | 7.8 |

# Life Table Construction: Survival Ratios

Finally, we have the life table survival ratio, denoted either as ${}_5P_x$ or ${}_5S_x$.

Survival ratio: ${}_5P_x$ or ${}_5S_x$

In symbols:

$$ {}_5P_x = \frac{{}_5L_{x+5}}{{}_5L_x} $$

Where:

${}_5L_x$ is as defined above.

# Life Table Construction: Survival Ratios

For survival from birth to ages 0 to 4 and for the open-ended age group, we have:

Survival ratio from birth to ages 0 to 4: ${}_5P_b$

$$ {}_5P_b = \frac{{}_5L_0}{5 \times l_0} $$

Where:

${}_5L_0$ and $l_0$ are as defined above.

Survival ratio of open-ended age group: $P_w$

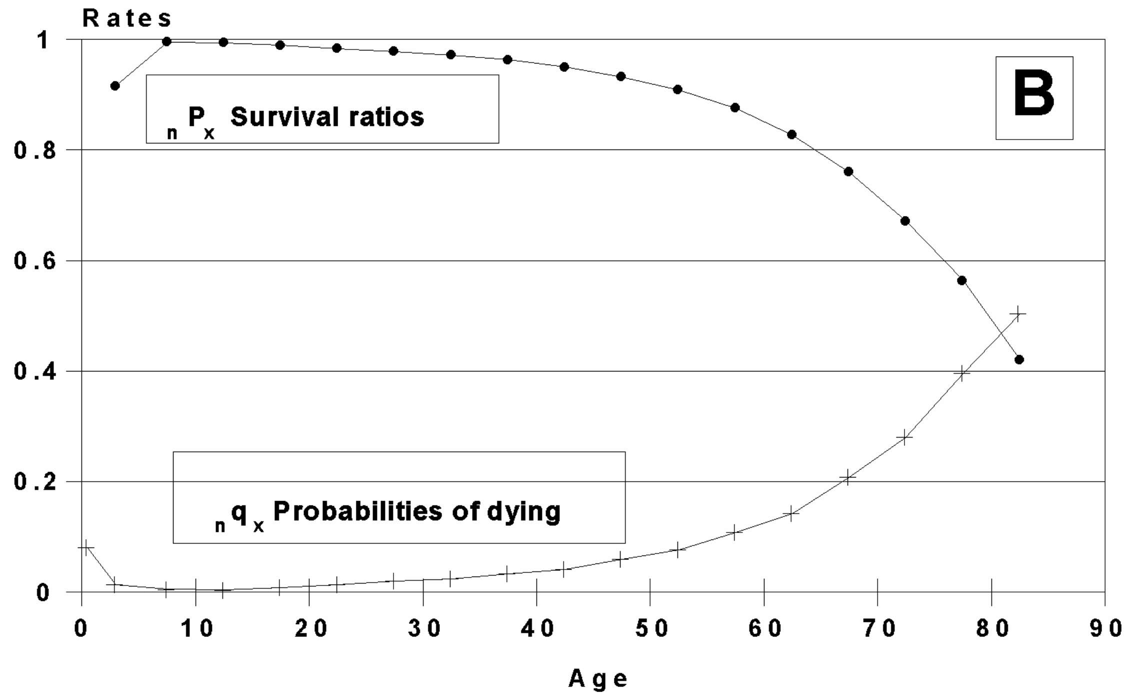
$$ P_{w-5} = \frac{T_w}{T_{w-5}} $$

Where:

T is as defined above; and
w represents the oldest age.

# Life Table Construction: Survival Ratios

Figure III-3. Selected Life Table Functions

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# Applications for Life Table Construction

- PAS: LTPOPDTH.xls
  - User enters population and deaths by ages
  - Allows for alternative infant mortality input
  - Allows for smoothing of ${}_nM_x$ values
- MORTPAK: LIFTB
  - User enters ${}_nM_x$, ${}_nq_x$, or ${}_nl_x$
- PAS: LTMXQXAD
  - User enters ${}_nM_x$ or ${}_nq_x$
  - Allows users to incorporate adjustment factors to correct nMx or nqx

# Exercises

- Construct a life table for your country based on population and death data by age using LTPOPDTH.

# Exercises

- (Optional) Use another application and compare the results to see why the results may be different.