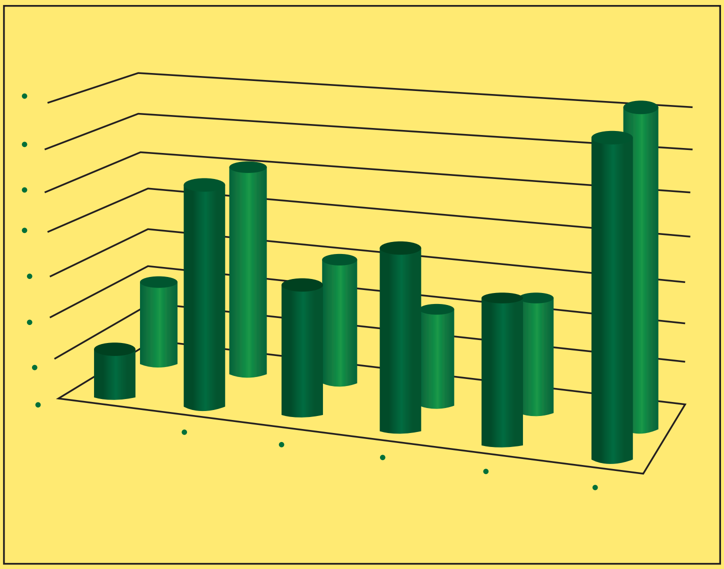
المعهد العربي للتدريب والبحوث الإحصائية

مجلة
العلوم الإحصائية



العدد رقم 22

مجلة علمية محكمة

يصدرها المعهد العربي للتدريب والبحوث الإحصائية

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ص.ب: 851104 عمان 11185 الأردن P.O. Box: 851104 Amman 11185 Jordan
Tel: +96265549805 Fax: +96265549804 www.aitrs.org info@aitrs.org

شروط النشر في مجلة العلوم الإحصائية

1 – تنشر المجلة البحوث والدراسات العلمية في المجالات الإحصائية والمعلوماتية المكتوبة باللغة العربية والانكليزية والفرنسية على أن لا يكون البحث المقدم للنشر قد نشر او قدم للنشر في مجلات او دوريات أخرى او قدم ونشر في دوريات لمؤتمرات أو ندوات.

2 – ترسل البحوث والدراسات الى أمين التحرير على أن تتضمن اسم الباحث او الباحثين وألقابهم العلمية وأماكن عملهم مع ذكر عنوان المراسلة وأرقام الهواتف والبريد الالكتروني. وان يرسل البحث المراد نشره الكترونيا (على قرص او بالبريد الالكتروني) وفق المواصفات أدناه:

أ – أن يكون مطبوعاً على ورق حجم A4 وان يكون على شكل عمود واحد ويستخدم للغة العربية نوع حرف (Simplified Arabic) و(Times New Roman) للإنجليزية والفرنسية وبحجم خط (12). وباستخدام Microsoft Word وعلى وجه واحد للورقة.

ب – الهامش مسافة 2.5 سم لجميع جوانب الورقة.

ج – يرفق الباحث ملخصاً عن بحثه باللغتين العربية والانجليزية والفرنسية بما لا يزيد عن صفحة واحدة.

د - يتم الإشارة الى المصادر العلمية في متن البحث وفي نهايته، مع مراعاة أن لا يتضمن البحث سوى المصادر التي تم الإشارة إليها في المتن ووفق الأصول المعتمدة في ذلك (اسم المؤلف، سنة النشر، عنوان المصدر، دار النشر، البلد).

هـ– ترقم الجداول والرسوم التوضيحية وغيرها حسب ورودها في البحث، كما توثق المستعارة منها بالمصادر الأصلية.

و– أن لا يزيد عدد صفحات البحث اوالدراسة عن (25) صفحة.

3 – يتم إشعار الباحث باستلام بحثه خلال مدة لا تتجاوز يومين عمل من تاريخ استلام البحث.

4 – تخضع كافة البحوث المرسلة الى المجلة للتقييم العلمي الموضوعي ويبلغ الباحث بالتقييم والتعديلات المقترحة إن وجدت خلال مدة لا تتجاوز اسبوعان من تاريخ استلام البحث.

5 – لهيئة تحرير المجلة الحق في قبول او رفض البحث ولها الحق في إجراء أي تعديل او إعادة صياغة جزئية للمواد المقدمة للنشر بما يتماشى والنسق المعتمد في النشر لديها بعد موافقة الباحث.

6 – يصبح البحث المنشور ملكاً للمجلة ولا يجوز إعادة نشره في أماكن أخرى.

7 – تعبر المواد المنشورة بالمجلة عن آراء أصحابها، ولا تعكس وجهة نظر المجلة او المعهد العربي للتدريب والبحوث الإحصائية.

8 – ترسل البحوث على العنوان الالكتروني للمجلة:

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المحتويات

Some methods for estimating the distribution of beta expanded with the application

Hajer Falah Taheer & Dr. Rawaa Salh Al-Saffar
Statistics department / College of Administration & Economics
Mustansiriyah University

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Some methods for estimating the distribution of beta expanded with the application

Hajer Falah Taheer & Dr. Rawaa Salh Al-Saffar
Statistics department / College of Administration & Economics
Mustansiriyah University

Abstract
A new extended beta distribution (ECB) with two parameters (shape parameters) is introduced. The expanded distribution is a special case of the beta distribution as it belongs to the family of exponential distributions. Exponential expansion is a method adopted by adding a new parameter to the exponent of the cumulative function. The survival rate function and the risk rate were also obtained for this distribution. Finding some mathematical properties, and by using some methods, including the least squares method and the weighted least squares method, to estimate the parameters using the Newton Raphson numerical algorithm, a Monte Carlo simulation study was conducted to evaluate the performance of the estimated parameters.

Keywords: beta distribution, expanded beta distribution, extension method, risk and survival function, Weigh Least Square, Least Square Developed.

مستخلص

تم تقديم توزيع بيتا موسع جديد (ECB) مع معلمتين (معلمات الشكل). التوزيع الموسع هو حالة خاصة لتوزيع بيتا لأنه ينتمي إلى عائلة التوزيعات الأسية. التوسع الأسي هو طريقة يتم اعتمادها عن طريق إضافة معلمة جديدة إلى أس الدالة التراكمية. كما تم الحصول على دالة معدل البقاء على قيد الحياة ومعدل الخطر لهذا التوزيع. إيجاد بعض الخواص الرياضية، وباستخدام بعض الطرق ومنها طريقة المربعات الصغرى وطريقة المربعات الصغرى المرجحة، لتقدير المعلمات باستخدام خوارزمية نيوتن رافسون العددية، تم إجراء دراسة محاكاة مونت كارلو لتقييم أداء المعلمات المقدرة

الكلمات المفتاحية: توزيع بيتا ، توزيع بيتا الموسع ، طريقة التمديد ، وظيفة المخاطر والبقاء على قيد الحياة ، الوزن المربع الأصغر ، المربع الأقل تطورا

1-Introduction

The primary goal of building a statistical probabilistic model is to determine the appropriate model that describes survival data or a group of data obtained from studies or experiments, etc. in a flexible and accurate manner. However, there is no single probability distribution suitable for the different data set, resulting in In order to expand the current classical distributions or develop new ones, in **2017 (Paula, F.V.,**

et al)[7] presented an expanded cardioid model, which is called the exponential cardioid distribution, some of its mathematical properties were derived, and two methods were used to estimate the parameters, in **2018 (Aydin, D.)**[5]**.** The statistics of the new distribution, and the parameters of the distribution were estimated using different methods, in **2021** presented **(Hassan, A. S. and Elgarhy, M.)**[13] a new extended family of the family of distributions that were created by Whipple, the properties of the extended Whipple family were studied, providing some special models in the established Whipple distributions as well as applying the derived properties of the extended family to these selected models, and several methods were used to estimate the parameters, In 2022 (Zohreh, Z., et al)[15] presented a new distribution called the expanded exponential gene distribution, the mathematical properties of the new distribution were derived, and different methods were used to estimate the parameters, in 2023 (Maysam, S, K.)[12] presented the distribution of the expanded exponential power function EEPF with four parameters, by the method of exponential expansion using an expanded distribution of the power function, and some mathematical properties were found, then we used the method of least squares to find the parameters.

2- The special case distribution of an extended beta distribution (ECB) (17,18)

The special case distribution of the beta distribution was used to explain the finite scarcity data sets, and this distribution is preferred for the best fit compared to the exponential distribution, the Weibull distribution, the lognormal distribution and other distributions because of its simplicity and applicability, and we will also use an expanded distribution from the special case distribution of the beta distribution And by means of this distribution, a new distribution will be created, and the probability density function PDF for the expanded distribution (ECB) is given in the following form:

$$f(t,\theta,\gamma) = \theta\gamma t^{\theta\gamma-1} \qquad 0 < t < 1 \; ; \quad \gamma,\theta > 0 \qquad \ldots..(1)$$

Where (γ,θ) represents the shape parameter, and the cumulative distribution function (CDF) is given by the following formula:

$$G(t) = (t)^{\theta} \qquad 0 < t < 1 \qquad \ldots\ldots(2)$$

A new distribution can be created using the exponential expansion method by adding a new shape parameter to the exponent of the cumulative function of the distribution, the special case of the beta distribution, and thus we can derive a new distribution that contains two proxy parameters:

$$F(t) = [G(t)]^{\gamma} \qquad \ldots\ldots(3)$$

Substituting formula (2) into (3), we get the following:

$$F(x) = \left[(t)^{\theta}\right]^{\gamma}$$

$$F(t) = [t]^{\theta\gamma} \qquad 0 < t < 1 \qquad \ldots\ldots(4)$$

By deriving formula (4), we obtain the probability density function for the new expanded exponential distribution (ECB) as follows:

$$f(t,\theta,\gamma) = \theta\gamma t^{\theta\gamma-1} \qquad 0 < t < 1 \ ; \quad \gamma,\theta > 0 \ldots\ldots(5)$$

The pdf function of the expansion distribution has two parameters (γ, θ) that represent the shape parameter.

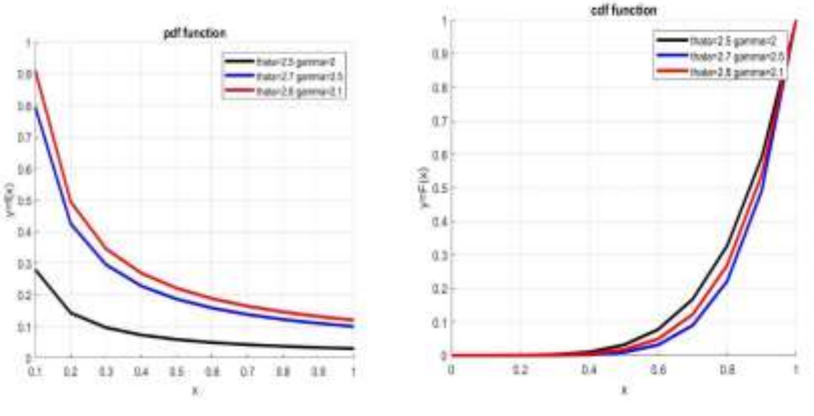

Figure (1-1) shows the CDF & PDF function for the new distribution (ECB))

3- Properties of an expanded distribution

After obtaining the new probability distribution (ECB), it is necessary to be acquainted with some of its characteristics:

1- The average

$$E(t) = \frac{\theta\gamma}{\theta\gamma + 1} \qquad \ldots\ldots(6)$$

2- Contrast

$$E(x - Ex)^2 = Var(x) = \frac{\theta\gamma}{(\theta\gamma + 2)(\theta\gamma + 1)^2} \qquad \ldots\ldots(7)$$

3- Standard Deviation

$$\sigma = \frac{\sqrt{\theta\gamma}}{(\theta\gamma + 1)\sqrt{\theta\gamma + 2}} \qquad \ldots..(8)$$

4- Coefficient of difference

$$C \cdot V = \frac{\sqrt{\theta\gamma}}{\theta\gamma\sqrt{\theta\gamma + 2}} * 100 \qquad \ldots..(9)$$

4-Survival Function (1,14,21)

It is defined as the probability that the patient will survive is greater than the specified time, and it is also considered one of the important statistical functions, due to its importance in medical and other fields, and the survival function can be expressed in the following formula:

$$S(t) = P_r(T \geq t) = \int_t^{\infty} f(u)du \qquad \ldots..(10)$$

$$= 1 - P_r(T \leq t)$$

$$S(t) = 1 - F(t) \qquad \ldots..(11)$$

Substituting formula (4) into formula (11), we get the general form of the survival function of the new distribution (ECB) as follows:

$$S(t) = 1 - t^{\theta\gamma} \qquad \ldots..(12)$$

Where

T: time of death

t: is the time between two time periods.

5- Failure Or Hazard Function (1,2,4,14)

It is defined as the percentage of failure or failure, which is the probability of failure occurring in the subsequent period of time, given that this item was in good condition, and is expressed as follows:

$$h(x) = \frac{f(x)}{S(x)} \qquad \ldots..(13)$$

Substituting formula (5) and (12) into formula (13), we get the general formula for the new distribution failure rate function (ECB) as follows:

$$h(t) = \frac{\theta\gamma t^{\theta\gamma - 1}}{1 - t^{\theta\gamma}} \qquad \ldots..(14)$$

The survival function is inversely proportional to the failure rate, and the failure rate function is directly proportional to time, as it increases with time.

The distribution is said to be (IFR) (i.e. increasing failure rate) if h(x) is a non-decreasing function of x, and it is said to be (DFR) (i.e. decreasing failure rate) if h(x) is not increasing for x.

6-Least Square Developed (LSD)[9]:-

This method was proposed by the researchers (Swain, Venkatraman and Wilson) and it is also called the Regression Procedure method.

Where we suppose ($t_1,t_2,\ldots\ldots\ldots,t_n$) represents a random sample with a specific distribution $F(t_i)$ and that t_i represents the ordered statistics of the sample values.

Whereas

$$E[F(t_i)] = \frac{i}{n+1} \qquad \ldots(15)$$

$$Var[F(t_i)] = \frac{i(n-i+1)}{(n+1)^2(n+2)} \qquad \ldots(16)$$

And using the expectation we can get the Least Squares Estimator (OLS) and my agency

$$\sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} \left[F(t) - \frac{i}{n+1}\right]^2 \qquad \ldots..(17)$$

And by substituting the (CDF) function for the new distribution into formula (17), we get

$$\sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} \left[t^{\gamma\theta} - \frac{i}{n+1}\right]^2 \qquad \ldots(18)$$

In addition, by deriving formula (18) to find the estimators of the parameters for the new distribution, my words:

$$\frac{d\sum_{i=1}^{n} {e_i}^2}{d\gamma} = \sum_{i=1}^{n} \left[\left(t^{\gamma\theta} - \frac{i}{n+1}\right)\left(\theta t^{\gamma\theta} lnt\right)\right] \qquad \ldots(19)$$

$$\frac{d\sum_{i=1}^{n} {e_i}^2}{d\theta} = \sum_{i=1}^{n} \left[\left(t^{\gamma\theta} - \frac{i}{n+1}\right)\left(\gamma t^{\gamma\theta} lnt\right)\right] \qquad \ldots(20)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\gamma^2} = \sum_{i=1}^{n} \left[\left(t^{\gamma\theta} - \frac{i}{n+1}\right)\left(\theta^2 t^{\gamma\theta}(lnt)^2\right) + \left(\theta^2 t^{2\gamma\theta}(lnt)^2\right)\right] \qquad \ldots(21)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\theta^2} = \sum_{i=1}^{n} \left[\left(t^{\gamma\theta} - \frac{i}{n+1}\right)\left(\gamma^2 t^{\gamma\theta}(lnt)^2\right) + \left(\gamma^2 t^{2\gamma\theta}(lnt)^2\right)\right] \qquad \ldots(22)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\gamma\theta} = \sum_{i=1}^{n} \left[\left(t^{\gamma\theta} - \frac{i}{n+1}\right)\left(\theta^2 t^{\gamma\theta}(lnt)^2 + t^{\gamma\theta} lnt\right) + \left(\theta^2 t^{2\gamma\theta}(lnt)^2\right)\right] \qquad (23)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\theta\gamma} = \sum_{i=1}^{n} \left[\left(t^{\gamma\theta} - \frac{i}{n+1}\right)\left(\gamma^2 t^{\gamma\theta}(lnt)^2 + t^{\gamma\theta} lnt\right) + \left(\gamma^2 t^{2\gamma\theta}(lnt)^2\right)\right] \qquad \ldots(24)$$

When the above derivatives are equal to zero, it is not possible to obtain accurate estimates when solving them by analytical methods. Therefore, we will use numerical methods to find estimates of the unknown parameters, which are (γ, θ), and one of these methods is the Newton Raphson algorithm method, which is given in the following formula:

$$[T_{n+1}] = T_n - [J_n]^{-1}[f_n] \quad \ldots\ldots(25)$$

where

n : the number of iterations (n=1,2,…………,k)

T: represents the parameter vector of the new ECB distribution

T_n: represents the estimates of the greatest possibility for iteration n

$T_{(n+1)}$: represents the estimates of the new parameters for the subsequent iteration (n+1)

f_n : represents the vector of the first derivative of the logarithm of the possibility function of iteration n

J_n : represents the second derivative matrix of the new distribution (ECB)

This method depends on the covariance and covariance matrix, so we find the second derivative of the parameters and partial derivatives as follows:

$$\frac{d^2 \ln Lf}{d^2\theta} = \frac{-n}{\theta^2} \quad \ldots\ldots(26)$$

$$\frac{d^2 \ln Lf}{d^2\gamma} = -\frac{n}{\gamma^2} \quad \ldots\ldots(27)$$

$$\frac{d^2 \ln Lf}{d\theta\, d\gamma} = \sum_{i=1}^{n} \ln t_i \quad \ldots\ldots(28)$$

Where

$$[f_n] = \begin{bmatrix} \frac{d \ln Lf}{d\theta} \\ \frac{d \ln Lf}{d\gamma} \end{bmatrix} \quad ; \quad [T_n] = \begin{bmatrix} \theta \\ \gamma \end{bmatrix} \quad ; \quad J_n = \begin{bmatrix} \frac{d^2 \ln Lf}{d^2\theta} & \frac{d^2 \ln Lf}{d\theta\, d\gamma} \\ \frac{d^2 \ln Lf}{d\theta\, d\gamma} & \frac{d^2 \ln Lf}{d^2\gamma} \end{bmatrix}$$

7-Weigh Least Square (WLS)[13] :-

This method works similar to the ordinary least squares method, where we also assume (x_1, x_2, … x_n) represents a random sample with a specific distribution $F(x_i)$ and that x_i represents the ordered statistics of the sample values.

Where

$$E[F(x_i)] = \frac{i}{n+1} \quad ; \quad Var[F(x_i)] = \frac{i(n-i+1)}{(n+1)^2(n+2)}$$

By using the expectation and variance, we get the weighted least squares (WLS) estimates, as follows:

$$\sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} \frac{1}{Var[F(x_i)]}\left[F(x) - \frac{i}{n+1}\right]^2 \quad \ldots (29)$$

By substituting the CDF function for the new distribution into formula (17), we get

$$\sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} \frac{i(n-i+1)}{(n+1)^2(n+2)}\left[x^{\gamma\theta} - \frac{i}{n+1}\right]^2 \quad \ldots (30)$$

Deriving formula (18) to find the estimators of the parameters for the new distribution, my words:

$$\frac{d\sum_{i=1}^{n} {e_i}^2}{d\gamma} = 2\sum_{i=1}^{n} \frac{i(n-i+1)}{(n+1)^2(n+2)}\left[\left(x^{\gamma\theta} - \frac{i}{n+1}\right)\left(\theta x^{\gamma\theta} lnx\right)\right] \quad \ldots (31)$$

$$\frac{d\sum_{i=1}^{n} {e_i}^2}{d\theta} = 2\sum_{i=1}^{n} \frac{i(n-i+1)}{(n+1)^2(n+2)}\left[\left(x^{\gamma\theta} - \frac{i}{n+1}\right)\left(\gamma x^{\gamma\theta} lnx\right)\right] \quad \ldots (32)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\gamma^2} = 2\sum_{i=1}^{n} \frac{i(n-i+1)}{(n+1)^2(n+2)}\left[\left(x^{\gamma\theta} - \frac{i}{n+1}\right)\left(\theta^2 x^{\gamma\theta} (lnx)^2\right) + \left(\theta^2 x^{2\gamma\theta} (lnx)^2\right)\right] \quad \ldots (33)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\theta^2} = 2\sum_{i=1}^{n} \frac{i(n-i+1)}{(n+1)^2(n+2)}\left[\left(x^{\gamma\theta} - \frac{i}{n+1}\right)\left(\gamma^2 x^{\gamma\theta} (lnx)^2\right) + \left(\gamma^2 x^{2\gamma\theta} (lnx)^2\right)\right] \quad \ldots (34)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\gamma\theta} = 2\sum_{i=1}^{n} \frac{i(n-i+1)}{(n+1)^2(n+2)}\left[\left(x^{\gamma\theta} - \frac{i}{n+1}\right)\left(\theta^2 x^{\gamma\theta} (lnx)^2 + x^{\gamma\theta} lnx\right) + \left(\theta^2 x^{2\gamma\theta} (lnx)^2\right)\right] \quad \ldots (35)$$

$$\frac{d^2\sum_{i=1}^{n} {e_i}^2}{d\theta\gamma} = 2\sum_{i=1}^{n} \frac{i(n-i+1)}{(n+1)^2(n+2)}\left[\left(x^{\gamma\theta} - \frac{i}{n+1}\right)\left(\gamma^2 x^{\gamma\theta} (lnx)^2 + x^{\gamma\theta} lnx\right) + \left(\gamma^2 x^{2\gamma\theta} (lnx)^2\right)\right] \quad \ldots (36)$$

Since the relationship is nonlinear between the equations, so we will use the Newton Raphson algorithm as in ordinary least squares.

8- Simulation

Simulation known as an optimal method for solving many complex problems that are difficult to solve in real life, including complex mathematical operations or the difficulty of providing real data when studying a particular phenomenon. One of the basic principles of simulation is to develop a program that represents or resembles the behavior of real reality using a computer, and often this reality is very

complex in representation with a calculator program, and despite that, the simulation method can give clear information about the reality that it represents.
The Monte Carlo method is one of the most important of these methods, which is considered the most common and used in researching and analyzing parameter estimators in several ways for the model under study.

- **stages of simulation**
  1- Setting the initial values and this stage is very important for the other stages to depend on.
  2- Random observations (data) are generated, which follow the new expanded beta distribution (CEB) represented by two parameters (γ,θ).
  3- The model parameters of the new distribution are estimated by the least squares(LS) method and the weighted least squares(WLS) method.
  4- To reach the best estimate of the function of survival and failure (risk), the Mean Squares Integral Error (MISE) standard and the MSE standard are used.

$$MSE(\hat{\beta}) = Var(\hat{\beta}) + \{Bias(\hat{\beta})\}^2 \quad \ldots..(37)$$

Where
$\hat{\beta}$: Estimation of model parameters($\hat{\beta} = \hat{\gamma}, \hat{\theta}$)
$Var(\hat{\beta})$: Variance of the estimator
$Bias(\hat{\beta})$: The bias estimator, which is the difference between the true value and the estimated value

$$MISE\left(\hat{S}(x)\right) = \frac{1}{r}\sum_{i=1}^{r}\left[\frac{1}{n_x}\sum_{j=1}^{n_x}\left(\hat{S}(x) - s(x_j)\right)^2\right] \quad \ldots..(38)$$

$$MISE\left(\hat{h}(x)\right) = \frac{1}{r}\sum_{i=1}^{r}\left[\frac{1}{n_x}\sum_{j=1}^{n_x}\left(\hat{h}(x) - h(x_j)\right)^2\right] \quad \ldots..(39)$$

That:
r: the number of iterations of the experiment (1000) times.
nx: the number of data generated for each sample.
Ŝ(x),ĥ(x)the estimated survival and failure function, respectively.
s(xj),h(xj): survival and failure function according to the initial values, respectively.

Table (1) represents the simulation results of the estimation methods (LS, WLS) for the expanded beta distribution (γ = 0.9, θ = 1.2)

N	Method	MSE		MISE(S)	MISE(h)
		$\hat{\gamma}$	$\hat{\theta}$		
25	LS	0.042025	0.042025	0.0057452	2.2953
	WLS	0.11261	1.44	0.62391	2.291
BEST		LS	LS	LS	WLS
50	LS	0.042025	0.042025	0.0018407	9.8685
	WLS	0.0047187	0.15181	0.0047499	5.0953
BEST		WLS	LS	LS	WLS
100	LS	0.019015	0.012383	0.0031321	1.8505
	WLS	0.10864	1.44	0.3483	0.35509
BEST		LS	LS	LS	WLS

Table (2) represents the simulation results of the estimation methods (LS, WLS) for the expanded beta distribution (γ = 1.7, θ = 1.5)

N	Method	MSE		MISE(S)	MISE(h)
		$\hat{\gamma}$	$\hat{\theta}$		
25	LS	0.029784	0.340528	0.009078	0.789204
	WLS	0.004232	0.013592	0.004867	9.44126
BEST		WLS	WLS	WLS	LS
50	LS	0.005703	0.019507	0.003655	0.117441
	WLS	0.042932	0.029634	0.003023	0.357779
BEST		LS	LS	WLS	LS
100	LS	0.00763	0.283462	0.000552	0.067932
	WLS	0.013896	0.004434	0.002898	0.587903
BEST		LS	WLS	LS	LS

Table (3) represents the simulation results of the estimation methods (LS, WLS) for the expanded beta distribution (γ = 1.7, θ = 2).

N	Method	MSE		MISE(S)	MISE(h)
		$\hat{\gamma}$	$\hat{\theta}$		
25	LS	0.00679	0.015304	0.006227	2.397784
	WLS	0.03984	0.031184	0.002005	1.532176
BEST		LS	LS	WLS	WLS
50	LS	0.01631	0.087478	0.001829	5.179061
	WLS	0.17203	0.12863	0.002022	0.143674
BEST		LS	LS	LS	WLS
100	LS	0.01904	0.092098	0.001723	1.744242
	WLS	0.22784	0.1694	0.001168	1.796181
BEST		LS	LS	WLS	LS

Table No. (1,2,3) shows the simulation results to estimate the new model coefficients (ECB) for three models of imposed parameters with three different sizes. The MSE criterion and the MISE criterion were used for the survival and risk function. We find that the lowest value of the MSE results was at N = 25 for the parameter $\hat{\gamma}$. As for the parameter $\hat{\theta}$, it had preference for the WLS method at N = 100, and we find the results of the mean squares integrative error MISE for the survival rate. The preference was at N = 100 for the WLS method, as it achieved the most number times better than the (LS) method, as well as for the MISE criterion for the risk rate, the preference was at N = 50 for the WLS method, as it achieved the most number of times better than the two methods (LS) and for two models (θ = 1.5, γ = 1.7, θ = 2) (γ = 1.7) for the parameters used, indicating a preference for parameter models at those values.

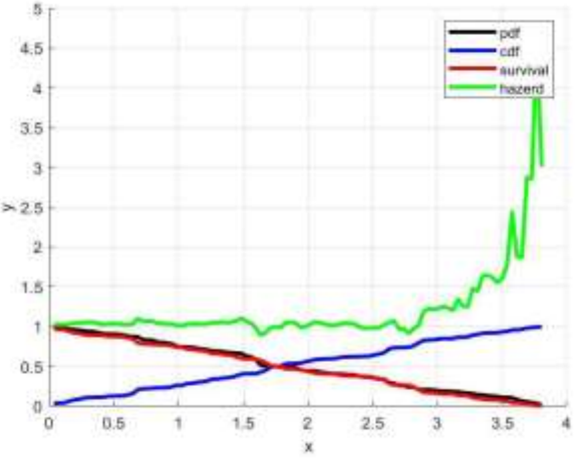

Figure (2) shows the PDF&CDF, survival and failure (LS,WLS) function of the ECB distribution.

9- Conclusions

A new distribution was created using one of the forms of the beta distribution, which contains one parameter (shape parameter), where we added a shape parameter using the exponential expansion, and thus a new distribution was obtained that belongs to the exponential distributions that contains two parameters of the shape, and the parameters of this distribution were estimated using the method Least squares and weighted least squares method The performance of these methods was evaluated using Monte Carlo simulation, where ECB values were evaluated in the volumes assigned to each model.

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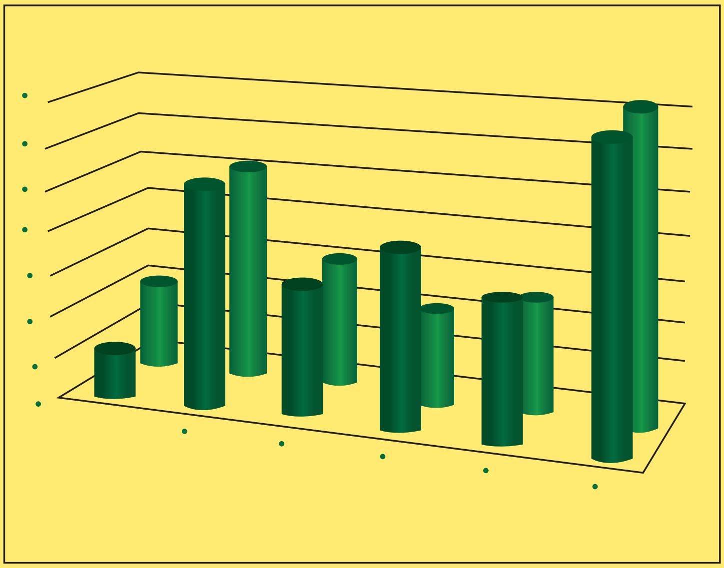

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