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تاريخ استلام البحث: 14 / 02 / 2022

تاريخ قبول البحوث: 28 / 03 / 2022

نشر البحث في العدد السابع عشر: ديسمبر / كانون اول 2022

رمز التصنيف ديوي / النسخة الالكترونية (Online): 2522-64X/658.5

رمز التصنيف ديوي / النسخة الورقية (Print): 2519-948X/658.5

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Abstract

The main purpose of control charts is to help determine if a process is stable and in-control, or unstable and out-of-control. When a process is stable, or "in control," this means that it is predictable and affected only by normal random causes of variation. An unstable or "out-of-control" process is affected by the same common causes of variation, but it is also affected by "special" or "assignable" causes.

This paper reviews the basic concepts on Statistical Quality Control (SQC) and its historical background. SQC techniques, when applied to measurement data, it can be used to highlight areas that would benefit from further investigation. These techniques enable the user to identify variation within their process. Understanding this variation is the first step towards quality improvement. There are many different statistical process control techniques that can be applied to data. The simplest statistical process control technique to implement is control charts. The purpose of this technique is to identify when the process is displaying unusual behaviour. In practice, it has been shown how to apply control charts of variables and attributes to the data from two xamples.

Keywords: Shewart's chart, statistical quality control, variables control charts, attributes control charts

مخططات مراقبة الجودة الإحصائية

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المستخلص

الهدف الرئيس من مخططات الجودة (السيطرة) هو المساعدة في تحديد ما إذا كانت العملية مستقرة وخاضعة للسيطرة، أو غير مستقرة وخارجة عن السيطرة. عندما تكون العملية مستقرة، أو "تحت السيطرة"، فهذا يعني أنها يمكن التنبؤ بها وأنها تتأثر فقط بالأسباب العشوائية الاعتيادية للتغير. تتأثر العملية غير المستقرة أو "الخارجة عن السيطرة" بنفس الأسباب الشائعة للتباين، ولكنها تتأثر أيضًا بأسباب "خاصة" أو "قابلة للتخصيص".

تستعرض هذه الورقة المفاهيم الأساسية لمراقبة الجودة الإحصائية (SQC) وخلفيتها التاريخية. عندما يتم تطبيق تقنيات (SQC) على البيانات المقاسة، فإنه يمكن استخدامها لتسليط الضوء على المجالات التي قد تحتاج إلى مزيد من التحقيق. هذه التقنيات تمكن المستخدم من تحديد الاختلاف في العملية الخاصة بهم. فهم هذا الاختلاف هو الخطوة الأولى نحو تحسين الجودة. هناك العديد من تقنيات التحكم في العمليات الإحصائية المختلفة التي يمكن تطبيقها على البيانات. إن أبسط تقنيات التحكم في العمليات الإحصائية التي يتم تنفيذها هي مخططات الجودة. الغرض من هذه التقنية هو تحديد متى إظهار العملية سلوكًا غير عادياً. من الناحية العملية تم بيان كيفية تطبيق مخططات السيطرة للمتغيرات والصفات من خلال بيانات مثالين.

الكلمات المفتاحية: مخطط شيوارت، مراقبة الجودة الإحصائية، مخططات التحكم في المتغيرات، مخططات التحكم في السمات

1. Introduction:

The term “quality control” consists of two parts: “control”, which is the procedures that are taken to ensure that the product or services conform to the specified specifications, while “quality” is the ability of the product or service to meet or exceed consumer expectations. It is the sum of the measures taken in various ways and methods to ensure that the product or service conforms to the specifications specified in advance to ensure that it can be accepted by the consumer in order to study and improve the production or service and control its costs. Productivity and service and in all fields: economic, agricultural, industrial ... etc.

There are eight dimensions of quality control:

- **Performance:** means whether the product performs the required function.
- **Reliability:** means how and when the product fails to perform its function.
- Durability: which is the duration (life) of the product or service.
- **Aesthetics:** refers to the form of the product or service..
- **Serviceability:** how easy is it to repair the product.
- **Features:** indicates the capabilities of the product.
- **Quality:** is the reputation of the product or company producing.
- **Standards Matching:** it is the suitability of the product or service to the standard design specified for it.

1.1 Statistical Quality Control (SQC):

It is one of the applications of statistics in practical life, especially in industry. It is defined as the organizational process of events that measure the actual performance of quality and its conformity with the standard specifications and then take the necessary corrective measures for these events wherever they are found. Thus, it can be said that quality control over production cannot be considered a process that aims to achieve a formula of congruence between the product and the standard specifications to indicate deviation from it or to exceed the permissible range only, but it is a wider process than that when a preventive or warning control is alerting to the defect before its occurrence[1]. The quality control over production is divided into two parts: controlling the quality of the produced material and controlling the production process. Whereas, the first part is concerned with classifying the material (primary or produced) using statistical inspection methods into acceptable and unacceptable methods, while the second part means keeping the process

in a state of statistical control, i.e. producing a high percentage of materials of acceptable quality, and in the event that any defect appears in the process, that defect is revealed. This means that quality control is a decision-making process regarding the progress of the production process.

1.2 Historical Background:

There is no doubt that the history of statistical quality control extends to the history of the industry itself. In the middle ages, quality control was regulated by a large degree of training and for long periods of time for craftsmen, and this was the basis for workers' pride in their products[2]. The use of statistical methods to control quality began in the United States of America where in 1791 the US Secretary of the Treasury, Alexander Hamilton, emphasized the necessity of using the sampling method in controlling the quality, according to the report he submitted regarding industries, where he explained in the ninth paragraph of the "importance of using inspection method, as this helps to promote the product and develop industries for the better, which leads to the protection of the producer and the consumer". In 1924, the American researcher Shewhart, who was also working in Bell Laboratories for the telephone, provided an optical device called by him the qualitative control charts, using the theory of probability distributions, through a memorandum prepared for this purpose, dated May, 16, 1924, to solve the problems and bottlenecks that occurred in Bell Laboratories for phones, that is why he is the first to use statistical methods in the field of quality control[3]. Shewhart in 1925 published a paper in which he demonstrated the importance of using statistical methods as an aid in preserving the quality of industrial products, which is a relatively new field in the use of statistical methods. In his research, he dealt with this important economic role that they play in preserving product quality and disclosing deviations and their treatment [4]. The use of quality control methods to a large extent in America during and immediately after the war, where Bell Laboratories sent a delegation to Washington to develop a sample examination program for army sites where a manual was published for sample testing used in the field of the Navy by a group of statistical researchers at Columbia University[5]. Page, in 1954 proposed a new quality control chart, which is the cumulative sum chart, symbolized by (Cusum)[6]. Barnard, in 1959 invented a mask similar to the V letter shape named as V-mask for the purpose of using it in

decision-making when using the Cusum-chart. This chart is the second development after the Shewart's chart. As this chart has received attention from researchers as it is more efficient than Shewart's chart. After that, in 1958 Roberts developed three other types of charts, he created two charts to control the rate and variations of quality, the first based on moving averages and the second on moving ranges. In 1959, Roberts presented the third chart, which relies on geometric moving averages[7]. Crosby, in 1979 demonstrated the importance of using statistical methods in quality control to obtain better quality, and this in turn leads to a reduction in the cost of production, because the better the quality of the product, the less the percentage of spoilage, and this leads to lower costs, i.e., he demonstrated that there is an inverse relationship between production quality and cost[8]. In 1980, Deming presented a paper entitled "Statistical Control of Quality", in which he showed how to use statistical methods in controlling quality. He is considered the pioneer in teaching these methods and holding courses in the United States of America[4]. In 1982, Deming demonstrated that the reason for the great success in Japanese industries and the production of high-quality goods is the extensive use of statistical qualitative control methods in production processes[4]. In 1983, Feigenbaum spoke about new concepts in organizing the function of quality control, stressing an important fact that quality control is the responsibility of all departments and is not limited to one section without the other to obtain good quality of the product[9].

As for what was stated by the Central Organization for Standardization and Quality Control in Iraq in 1983 regarding the requirements of quality control as the central body responsible for the quality of products in the Iraqi industry, it was represented in preparing a guide for integrated quality control in implementation of its tasks that come in the forefront of advancing the national economy and improving production in addition to the need to specify Iraqi standards and standards inspired by the Iraqi environment. The company's senior management organizes special studies to improve the quality while ensuring efficiency and cost savings through statistical methods of examination and analysis of results[10].

Most of the previous studies and research on the issue of quality control depend on one variable that reflects the quality of one characteristic of the quality of the product and the other variables are prepared by virtue of the constant, but there are cases that require controlling more than

one characteristic. In 1947, Hotteling used a multivariate method in the quality control using a T^2-chart in a book that includes some statistical methods used to control the quality in case that there are more than one variable (or characteristic) associated with each other and that depends on the distribution of multivariate and Hotteling is the first to use a multivariate method in the quality control[11]. In 1957, Jackson & Morris published an applied paper on the use of the multivariate statistical method to control the quality of photographic production using the ellipse cart. In their research, they treat the case of three correlated variables that reflect the quality of the product photography[12].

The first efforts to study quality in the medical field and in industry began at the beginning of the twentieth century. One of the pioneers of quality control in medicine was the famous osteopath Ernest, A. Codman[13]. So health institutions are looking for quality products and seeking to improve the quality of their services by relying on clinical and operational activities such as (laboratories, radiology, etc.). In 1991, Schaake et al demonstrated the importance of studying the qualitative control of treatment methods with toxic drugs and deep radiation therapy for cancer patients in several medical centers and at different stages of the disease, and reached the importance of qualitative control in improving treatment outcomes by reducing drug side effects and thus reducing growth Tumors and improving the final outcome of treatment in a better way[14]. In 1998, Polygenis et al studied the relationship between alcohol use and the occurrence of birth defects in newborns using specific control. He took a sample consisting of (130,810) cases of which (24,000) used alcohol and the rest did not use alcohol, and depending on the quality control it was found that there is no relationship between alcohol use and the occurrence of birth defects[15]. In 2000, Kouri et al evaluated the degree of specificity of the laboratory tests, where he used the urine tests based on the upper limit of the normal values of these tests using the specific control and the accuracy of these analyzes were evaluated depending on the sensitivity and specificity of these analyzes by setting a grace period determined to the minimum which is the discovery of the presence of the substance in low concentrations in the urine and the upper limit, which is to establish its existence conclusively and for sure[16].

Another field in which statistical quality control has been used and applied is clinical chemistry and hematology, in 2001, Westgard

introduces the Six Sigma theory in clinical chemistry as another way of establishing quality specifications. This theory had already been tested in the industry for almost 20 years, with great success in reducing the number of defective products manufactured. A few years later (2003), a new standard was created exclusively for medical laboratories: the EN ISO 15189, which was revised in 2007 and 2012[17]. In 2007, Marshall and Mohammed used control charts to monitor mortality rates after coronary artery bypasses[18]. More recently, Urdinola and Rojas-Perilla in 2013 proposed to use this approach to identify the under-registration of adult mortality in Colombia[19]. In 2018, STC is widely used in Geman's Hospitals, External quality assurance (EQA) in Germany is regulated by the Directive on Measures concerning Quality Assurance in Hospitals. According to this directive, each patient's quality of treatment is reflected in a set of nationally standardized performance indicators. The raw case based performance data are transmitted to the regulatory agency by the end of February following the reporting year. The annual mean of the performance indicator is then compared to the national target, and, if relevant deviations are detected, appropriate interventions are initiated[20]. Motivated by the coronavirus disease 2019 (covid-19) pandemic, Perla & et al in 2020 developed a novel Shewhart's chart to visualize and learn from variation in reported deaths in an epidemic. During a pandemic, timely reliable signals that an epidemic is waxing or waning may have mortal implications. This novel chart offers a practical tool, accessible to system leaders and frontline teams, to visualize and learn from daily reported deaths during an epidemic. Without Shewharts' charts and, more broadly, a theory of variation in our epidemiological arsenal, we lack a scientific method for a real-time assessment of local conditions. Shewharts' charts should become a standard method for learning from data in the context of a pandemic or epidemic [21].

2. Material and Methods:

2.1 Deviations in the Production Process:

The deviations in the production process are divided into two parts: the first part is a random variations which represents the differences that cannot be traced back to a single reason, but to a large and complex group of reasons. These differences are due to chance and are usually not significant and do not affect the quality of the product as long as they fall within the permissible limits. But if these limits are exceeded, they constitute a threat to the production process, and these reasons cannot

be completely controlled and eliminated. As long as it is difficult to overcome, there is no need to make efforts to remove it, as this leads to cost and waste of time. The second part of deviations in the production process is deterministic variations which is due to actual causes called assignable causes in quality control due to the existence of an actual defect in one of the factors of production, in this case the process is out of control, and these causes can be controlled by identifying and studying them and working to remove them where its moral impact on the production process, so the responsible person must identify these reasons and try to eliminate or reduce them because their presence leads to the deviation of the production process from the specifications set for it. There are many reasons that lead to this type of deviations, including:

1. There are natural differences in the composition of the raw materials.
2. There is a difference in the level of skills of researchers and workers.
3. Significant differences in the test method or devices (equipment).
4. Neglect by workers in the production process and their inactivity.
5. Inaccuracy in reading measurements by an official.
6. The chronological age of the machines.
7. Lack of training for workers.
8. Change in temperature and humidity.

2.2 Inspection Policies:

There are many inspection policies that are carried out on the quality of the material produced in practical life to detect deviations and random changes, among which are[22]:

1. Inspection in terms of the producing units. These are divided into several inspection policies, including:
   - inspecting a unit from among each (n) units produced.
   - inspecting all produced units (100% comprehensive examination).
   - inspecting (n) units for each (N) units produced.
2. Inspection in terms of time:
   In this case, the units produced are examined during periods of time that may be regular or irregular, for example:
   - inspecting one unit (or sample of n units) at fixed and equal intervals (Fixed Interval Inspection).
   - inspecting (n) units for each (N) units produced in equal time periods.
   - inspecting a unit (or sample of n-unit size) in unequally spaced short periods of time.

2.3 Quality Control Charts:

Quality control charts are considered one of the scientific methods commonly used to control the quality of the product in order to obtain a better quality product according to the required specifications. Interest in this topic began at the beginning of the 18th century, and its use developed as a result of the development of statistical theory. These charts were invented by the American statistician (Dr. W.A. Shewhart) in the 1924, who was working in Bell Telephone Laboratories, where he used and developed statistical methods for the purposes of qualitative control such as frequency distributions, probabilities, and normal distribution. These charts were called by him the Quality Control Charts (QCC), after he published three studies in this field (1926-1927). The preparation of such charts is based on the use of one of the common statistical measures (indicators) such as the mean, range, or standard deviation[1]. The main objective of using these charts is to detect the actual abnormal changes in the parameters of the production process. On this basis, Shewart's goal of using charts was to keep the production process under control, that is, to produce a high percentage of acceptable materials, to detect any defects that occur in the production process, and to work on removing it, if any. Based on the foregoing, it can be said that the purpose of using charts is to control the mean (or level) of product quality or the changes and deviations in quality. The scientific idea behind using quality control charts lies in sampling theory and statistical inference in addition to normal distribution. Quality control charts are considered one of the most important common statistical methods used in quality control because they are easy to use as an alternative to using hypothesis testing because the production process is under control, and this is done by drawing random samples from the production line after determining the characteristic of the produced material to be controlled that reflects the quality of that material and then conducting statistical analysis of the data plotted on the chart.

In general, Shewart's chart consists of three lines that do not cut the horizontal axis, namely:

1- The center line, or called the target line, which represents the specified criterion for the level of required quality, symbolized by T.
2- The upper control limit of the chart represents the upper limit allowed for differences from the required level of increase, and it is symbolized by the UCL.

3- The lower limit of the chart represents the minimum permissible difference from the required level by decrease, symbolized by the LCL.

The two limits represent statistical tolerance limits, because no matter how accurate the production process is, there must be a difference between production units. If most of the points fall in the tolerance period between the two limits of control and are distributed normally around the target line, then the process is under statistical control, but when one or more points out of control means a defect in the process, and the cause must be discovered and removed. The position of the control limits on the chart is (± 3.96σ) from the target line, where (σ) represents the standard deviation (assuming it is constant and known) and to facilitate the calculations, Shewhart used control limits (± 3σ) from the target line and for this reason it was called the Shewhart's chart (3-sigma chart). When we add and subtract (± 3σ) from and to the target line results in a range that includes approximately (99.73%) of the distribution, and this is one of the characteristics of the normal curve, that is, if the random causes alone affect the process, this means that the probability (99.73%) of the values of the statistical indicators drawn on the chart fall within the limits of control and are distributed in a manner that follows the normal distribution around the target line. This means that it is possible (by chance) that one of the statistical indicators falls outside the limits of control without implying a defect in production with a probability of 0.27%, therefore, it is expected that (997) points out of a total of (1000) points fall in the tolerance period, at which time the process will be under control. The choice of control limits (± 3σ) to and from the target line gives an economic balance of the costs resulting from two types of statistical errors that may occur when deciding on the progress of the production process. Therefore, control limits (T ± 3σ) should be used unless they are strong practical reasons that prevent this, as in cases of controlling a quality that requires extreme accuracy of the quality of the produced material, it is possible in this case that the control limits fall within a distance (± 2σ, ± 1.5σ, or ± 1σ, …etc.) from the line target. Shewart's decision was that if all the points fall between the limits of control, the process will be under control, but if one point falls outside the limits, the process is out of control, and this means that there is a defect that has occurred or may occur later in the production process.

2.4 Statistical Errors:

There are two types of statistical errors that can be committed when making decisions about the production process. Type one error: this type of error occurs when we decide that the process is out of statistical control, but in fact it is under control, and this type of error is somewhat expensive because it leads to searching for the causes of the defect that made the productive process out of control, which in fact does not exist and it is possibility. When the error of the first type is small, the control limits are at a distance (± 3σ) from the target line, that is, the error of the first type with probability (α = 0.0027), this error is called false alarm. Type two error: this kind of error occurs when the decision is taken on the basis that the process is under control, but in fact it is out of control, meaning that the decision-maker about the process believes that the causes of deviations are random reasons for the fall of all points within the limits of control, but in fact the process is outside statistical control because there are reasons actual. It is not necessary for the process to be under control if all points fall within the limits of control because there are actual changes in the production process.

2.5 Modified Shewhart's Chart (MSC):

To this day, Shewharts' charts are known the Ordinary Shawhart's Chart (OSC), for their ease of use and simplicity, and the second reason is their ability and sensitivity to detect the significant and significant changes in the production process very quickly, but at the same time it suffers from some deficit because it relies on one point in making a decision, that is, it does not take the previous information into consideration, in addition to its inability to detect small and minor changes. Therefore, many researchers tried to overcome these weaknesses by modifying and developing the regular Shewarts' charts by drawing internal warning lines, usually placed at a distance (± 2σ) from the target line in addition to the control limits (LCL and UCL) for early warning before reaching the level quality to the lower and upper limits, usually one point out of a total of (20) drawn points or two points out of a total of (40) points located above or below the warning lines considered a normal condition, and more than that are considered abnormal and this means that a defect will occur in the production process in the near future. These are called modified Shewarts' charts according to increasing their ability and sensitivity to detect small changes in product quality.

2.6 Importance of using Quality Control Charts:

The use of quality control charts has great importance in the course of production processes by detecting any defect that occurs in the production process as quickly as possible and maintaining the continuity of the process under control, and this will give a good opportunity to develop the product in quality or as it helps to open new horizons for the development of the process mechanism productivity, making it easier and more flexible. Since this topic has many aspects, we will try to define the most important general points for the benefits of quality control charts[23]:

1. Monitor the production process and compare its actual performance with the specified specifications.
2. The distinction between the natural deviation resulting from the random causes inherent in the production process and the deviation resulting from the actual causes that can be easily identified and discovered for the purpose of eliminating them.
3. Early detection of errors and defects, and this helps reduce the percentage of defective units in production, and this in turn leads to a decrease in the total cost of quality.
4. It contributes to reducing the inspection process because it deals with examining part of the production (samples), and this helps reduce the costs of testing.
5. Determining the responsibility of the offender for the deviation or the error and the reasons for that deviation, in order to enable those responsible for production to address the deviation and prevent its occurrence or its recurrence later, as well as determine whether the cause is the production work system or a defect in the machines and production requirements or in the quality of the working cadres and finding solutions for each one of these reasons.
6. It gives a "continuous" perception of the position of change in quality during the course of the production process and the extent of its conformity with the specified specifications.
7. It has a preventive or warning role, alerting the defect before it occurs, and this preventive role improves the economics of production by warning about the occurrence of defects before they happen. Thus, losses resulting from product damage can be avoided.
8. Providing information and statistical data to reveal the development in improving the quality of product so that it can be used in the future.

9. Help in guessing production costs.
10. Play an "important" role as a moral incentive for employees and as a basis for evaluating performance and granting incentives. The charts are also an important tool for evaluating the production position of the administration.

2.7 Classification of Control Charts:

In general, the quality control charts are classified into two categories: control charts for variables and control charts for attributes, this classification depends on the characteristic of the produced material, which is divided respectively: measurable characteristic and non - measurable characteristic.

2.7.1 Variable Control – Charts:

If the quality of the product is measurable and is expressed in numbers, it is called a variable or quantitative, meaning that this characteristic can be measured quantitatively and is considered a "continuous" random variable. This characteristic is expressed in terms of the basic units (meter, kg, hour, amps, percentage, degree,…,etc). These chars are used to control the rate (level) of quality and the variation in the quality of the produced material, the most important of these charts are:

1. X-Chart (Single Value-Chart):

In many cases, there are few measurements of the quality characteristic to be studied due to the high cost, the time taken, or the scarcity of the examined parts. Therefore, there are production cases in which it is preferable to use a single observation for each time period for the purpose of monitoring. In this case, the individual values obtained are drawn on the chart. One of the advantages of this chart is its ease of personal use by the employees for ease of understanding it. It also gives information to the specified data while the chart (will be mentioned later) gives the information after a delay in time until sufficient information is available. As for its disadvantages, it requires a large number of observations to know the state of lack of control, and it does not summarize data as is the case in chart[1]. The target line for this chart represents the mean of all observations. The two limits of control are $\left(\overline{X} \pm 3S_X\right)$, the vertical axis represents the quality characteristic, while the horizontal axis represents the sequence of observations (or time).

2. $\overline{\mathbf{X}}$-Chart (Average– Chart):

One of the most widely used quality control charts is the average chart $\bar{X}$. This chart is used to control the quality mean and it is the first chart designed by Shewart. The target line for this chart represents the general mean of all observations ($\overline{\overline{X}}$) (the average of all means of the samples drawn). The control limits are $\left(\overline{\overline{X}} \pm 3\sigma_{\bar{X}}\right)$. One of the benefits of the chart is that it shows the change in the process quickly. Also, when drawing the average values, they are closer to the target line than the individual values, because when calculating the sample average, the effect of the outliers will decrease. Also, the average is a more efficient indicator than the individual values because it contains more information.

3. R-Chart (Range– Chart):

This Chart is used to control the variations in the quality of the produced material, that is, it helps to detect changes (differences) in the process. This chart is prepared from the ranges of the drawn samples, as the range for each sample is drawn against the sequence of samples (or time), and the range in each sample is measured by calculating the difference between the largest value and the smallest value for the observations of that sample. When the sample size is less than 10 (n <10), because when the sample size is greater than (10) or more, it is preferable to use the σ-chart (will be mentioned later) because $\hat{\sigma}$ contains more accurate information. The target line for this chart is the average of the sample ranges $R = \sum_{i=1}^{m} R_i/m$. The values drawn on this chart represent the values of the range of each sample, while the control limits are ($\bar{R} \pm 3S_R$), or it can be calculated directly by multiplying the average ranges by the coefficients of the two control limits as follows $UCL = D_4\overline{R}$ and $LCL = D_3\overline{R}$ where D_3 and D_4 are constants taken from the table of control limits[2]. In general, the limits of this chart are asymmetric if the sample size is $(n \leq 6)$ because the value of the control limits parameter is (D_3 = 0), meaning that the chart consists of only one term.

4. S-Chart (Standard Deviation– Chart):

This chart is also used to control variations and variances in the quality of the produced material. The range was used as an alternative to the standard deviation for its ease of calculation. However, the standard deviation of the sample is calculated by taking all the sample observations and not the largest value and the smaller value. Therefore,

is considered the most accurate and efficient of the range, it contains more information, but when the sample size is less than (10) observations, it is The R-Chart is preferred. The target line for this chart is the average of the sample standard deviation ($\bar{S} = \sum_{i=1}^{m} S_i/m$). The two control limits ($\bar{S} \pm 3\sigma_S$) can be calculated directly by multiplying the mean standard deviations by the coefficients of the control limits B_3 and B_4 as $UCL = B_4\bar{S}$ and $LCL = B_3\bar{S}$. Whereas, B_3 and B_4 are constants taken from the table of parameters of control limits[2].

5. MA-Chart (Moving Average– Chart):

The previous charts do not take into account the previous information when making a decision, i.e. the values of the observations are calculated for one time only, and this is considered a weakness for that charts, which motivated many researchers to develop them and find new charts to address this deficiency, one of these charts is the moving average chart. The goal of using this chart is the same as the goal of the $\bar{\bar{X}}$-Chart i.e. to control the average of the quality level, but here it is based on a moving basis (k) and there are two ways to calculate moving averages of the rank (k) for a group of observations $X_1, X_2, \dots, X_n$.

The first method: $\overline{X}_i^{(k)} = \dfrac{X_i + X_{i+1} + \cdots + X_{i+(k-1)}}{k}$; i = 1,2,..., N-k + 1

where the sequence of averages is in ascending order.

The second method: $\overline{X}_i^{(k)} = \dfrac{X_{i-k+1} + X_{i-k+2} + \cdots + X_i}{k}$; i = k, k + 1, ..., N

where the sequence of averages is in descending order.

The moving average values of the samples $\bar{X}_i^{(k)}$ (k represents the moving average rank) are plotted on the chart against the sample sequence (time). The target line for this chart represents the general average of the moving averages: $\bar{\bar{X}}^{(k)} = \sum_{i=1}^{N-k+1} \bar{X}_i^{(k)}/(N-k+1)$,

the ccontrol limits for this chart are:

$UCL = \bar{\bar{X}}^{(k)} + 3\sigma_{\bar{X}^{(k)}} = \bar{\bar{X}}^{(k)} + A_2\bar{R}^{(k)}$ and $LCL = \bar{\bar{X}}^{(k)} - 3\sigma_{\bar{X}^{(k)}} = \bar{\bar{X}}^{(k)} - A_2\bar{R}^{(k)}$

where $\bar{\bar{X}}^{(k)}$ it is the general average of all moving ranges, and A_2 is extracted from the table of two limit coefficients.

6. MR-Chart (Moving Range– Chart):

The purpose of using this chart is the same as using (R-Chart and S-Chart), this means that it is used for the purpose of controlling the variance in the quality of the produced material, but it depends on the moving base.

The moving ranges of order (k) are calculated for a group of observations $X_1, X_2, \dots, X_N$ by the following formula:

$R_i^{(k)} = \max(x_{i-k+1}, x_{i-k+2}, - - —, x_i) - \min(x_{i-k+1}, x_{i-k+2}, - - —, x_i)$; i = k, k + 1, …, N

The values of the moving range $\bar{R}_i^{(k)}$ are drawn on the chart against the sequence of samples (time). The target line for this chart represents the average of all moving ranges $\bar{R} = \sum_{i=1}^{k} \bar{R}_i^{(k)}/N$ while the control limits ($\bar{R}^{(k)} \pm 3S_{R(k)}$) or it can be calculated directly as is the case in the control limits of the R-Chart, that is $UCL = D_4\overline{R}^{(k)}$ and $LCL = D_3\overline{R}^{(k)}$.

7. GMA-Chart (Geometric Moving Average-Chart):

The purpose of using a GMA-Chart is the same as that of $\bar{X}$-chart, to control the quality rate. A geometric moving average is calculated by the following formula:

$Z_i = (1\text{-}r)\, Z_{i-1} + r\, X_i$; i = 1,2,…, N

where (X_i) represents the most recent observation and is given the weight (r) and (Z_{i-1}) represents the previous geometric moving average with the weight (1-r), r represents the smoothing constant, and the value of r is determined depending on the last observation, since its value is proportional to the importance of the last observation. If the last observation was significant, the value of r would be large and vice versa, it is found that the best value of r is (0 <r <0.3), especially (r = 0.2). The weights with time are always reduced because the old data are of little weight and the most recent observation is given more weight. The values of the geometric moving average (Z_i) are plotted on the chart against the sequence of observations, which is why the chart is named so. The target line is the average ($T=M_0=Z_0$) and the value of Z_0 is always taken from previous data and is represented by (in the case of time zero) and if the value of M_0 is not known, we find a value for the previous data. As for the limits of control, it is UCL= T + $3S_Z$ and LCL= T – $3S_Z$ whereas $S_Z = (r/2 - r)^{0.5}$, the value of σ is known or estimated by the sample size as indicated on the chart. We note that this chart took all the previous data into consideration and gave a weight to each observation whose effect fades with time and the largest weight given to the last viewing X_i.

8. Cusum-Chart (Cumulative Sum-Chart):

This type of technique was used for the first time in the field of quality control by the scientist Page in 1954 and was considered a turning point in the field of quality control. The aim of using this chart is to control the

quality rate of the product as in the $\bar{X}$-Chart and the MA-Chart and the GMA-Chart. The basis for forming this chart is the accumulated sum of errors (S_n), which is calculated by the following formula:

$$S_n = \sum_{i=1}^{n} (X_i - k) = \sum_{i=1}^{n} e_i$$ where $e_i = X_i - k$, X_i is the value of observation i taken from the production line, k is a selected parameter whose value is close to the value of the target line M_0 and is usually determined by previous experiences, and e_i is the error of X_i about the parameter k. One of the advantages of this chart is that it takes all the observations into consideration when making a decision. It also does not include a target line and no control limits as in the previous charts, but it contains an easy method in practice that is used with the chart to find out whether the process is under control and this method is called the mask and is in the form of a letter-V (V-Mask) invented by the British scientist Barnard in 1959 and it is transparent and symmetric. This mask is placed on a drawing of Cusum-Chart, so that the head of the mask is at a horizontal distance of (d) from the last (S_n) and at an angle of (θ) between each arm of the mask and the horizontal line between the head of the mask and the last S_n. If the accumulated curve S_n cuts one of the arms of the V-mask, this indicates a defect in the production process. Plotted on this chart the values of S_n (cumulative sum) against the sequence of samples, from which the name of this chart came.

2.7.2 Attributes Control – Chart:

In some cases, the quality of the product cannot be measured and expressed in known units of measurement, meaning that production units are described by certain unmeasured properties. These characteristics are called attributes. These can be seen by eye, touch, hearing and so on. The qualities are expressed as good or not good, conform to the specifications or non-conformity, and are expressed as discrete variables. The word "attributes" is a term used for qualitative control, which means a partial defect or a total defect, partial defects it is said that the unit contains a defective part or several defective parts such as air bubbles in pieces of glass,…etc. while defective means that if a complete unit is defective (not in conformity with the specification) such as TV, radio, machine,…,etc, in this case, the unit is called the total defective. The control charts depend on the qualitative characteristics on the percentage of defects in the drawn sample or the number of defects in the produced unit, and the most important of these charts used to

control the total defective (defective) are (P-Chart, np-Chart, 100% P-Chart) and these charts are based mainly on binomial distribution. As for the most important charts used to control the partial defect (c-Chart), this chart is based mainly on the Poisson distribution. There are other uncommonly used boards (D-Chart, U-Chart,…,etc.) [23].

1. P-Chart (Proportion of Defective-Chart):

This chart is used if the judgment on the quality of the tested unit is defective or not defective, meaning that the units are described with certain characteristics that are not measured. Each production process assumes the existence of acceptable and unacceptable units of production, and of course the defective units constitute a certain percentage of production. The aim of using this chart is to control the percentage of defective units P, as the number of defective units follows the binomial distribution. Regular estimation of the defective percentage for each sample shall be as $\hat{P}$ = number of defective units in the sample/ number of tested units in the sample. $\hat{P}$ Plots on the chart values for each sample against the sequence of samples (or time). The target line for this chart represents the average of the percentages of defective units for all samples ($\bar{\hat{P}}$) and may be taken from past data. It is calculated as follows: $\bar{\hat{P}} = \sum_{i=1}^{m} \hat{P}_i/m$ where m represents the number of samples drawn, preferably 25 or more, the control limits for this chart are $UCL = \bar{\hat{P}} + 3S_{\hat{P}}$ and $LCL = \bar{\hat{P}} - 3S_{\hat{P}}$ where $S_{\hat{P}}$ represents the standard deviation of the defective percentages and determines its value as follows:

1- If the size of the samples is equal (constant) then $S_{\bar{\hat{P}}} = \sqrt{\frac{\hat{P}(1-\hat{P})}{n}}$

2- If the size of the samples is variable, then the control charts will take the form of a zigzag line, and this means that each sample has control limits and are calculated as:

$S_{\bar{\hat{P}}} = \sqrt{\frac{\hat{P}(1-\hat{P})}{n_i}}$; *i=1,2,…,m*. There is another way to calculate $S_{\hat{P}}$ by finding the average of the number of samples drawn, so that: $\bar{n} = \sum_{i=1}^{m} n_i/\mathrm{m}$, $S_{\hat{P}} = \sqrt{\frac{\hat{P}(1-\hat{P})}{\bar{n}}}$

2. nP-Chart (Number of Defective Chart):

It is possible to deal with the number of defects rather than the percentage of defects. This type of chart is used for ease of understanding by the persons working in the production, as the test results are indicated directly on the chart without any calculations. The purpose of using this chart is to control the number of defective units in the production process as the number of defective units in the sample equals ($n\hat{P}$). The average of the number of defective units in the samples represents the target line for this chart ($T = n\overline{\hat{P}}$) and the control limits ($n\overline{\hat{p}} \pm 3\sigma_{n\hat{P}}$) where

$$\sigma_{n\hat{P}} = \sqrt{n\hat{P}\left(1-\hat{P}\right)} \quad , n\overline{\hat{p}} = \sum_{i=1}^{m} n\hat{p}_i / m$$

Plot on the chart the number of defective units per sample versus the sequence of samples.

Since the defective number is an integer number, therefore the values of the control limits must be integers while the target line value can be fractional.

3. 100 P%-Chart (Percentage Defective-Chart):

The goal of using this chart is the same as the P-Chart but by converting defective ratios into percentages. The values extracted from these charts are the same as those used in P-Chart, but multiplied by (100), so that the target line is (T = 100 P%). As for the two control limits, they are:

$$LCL = 100\,\overline{P}\% + (3)(100\%)\sqrt{\frac{\overline{P}\%\left(1-\overline{P}\%\right)}{n}} \text{ and}$$

$$UCL = 100\,\overline{P}\% - (3)(100\%)\sqrt{\frac{\overline{P}\%\left(1-\overline{P}\%\right)}{n}}$$

The converted values are drawn as a percentage.

4. C-Chart (The number of defects per unit Chart):

This chart is considered the second type of quality characteristics charts and is used in the event that it is not possible to draw samples from the production line, for example (the presence of air bubbles in the glass plate, the presence of errors in the typesetting of the printer characters in a printed page, the presence of oil stains on the paper,…,etc.). In this case, it is preferable to take units that each unit represents a complete sample (the sample size is not specified or is very large) and then divides this unit into small squares, one of which cannot accommodate more than one defect (if any). The purpose of using this chart is to control the defects of

each unit, and the number of defects per unit (c) follows an approximate Poisson distribution for the following reasons:

1. It is used with rare accidents because the number of defects in one unit is very small in relation to the number of very many squares.
2. Each square either contains one defect, or it does not contain, in the language of probabilities, the probability of more than one event occurring in any small period of time Δt equals zero, so the probability of having more than one defect in the square is equal to zero, and this is one of the properties of the Poisson distribution.

The target line for this chart represents the overall average of all defects in the tested production units i.e. that:

$$T = \bar{c} = \frac{\sum_{i=1}^{m} c_i}{m}$$ whereas c_i = number of defects in unit i and m = number of examined units

As for the two control limits $\bar{c} \pm 3\sqrt{\bar{c}}$, they draw on this chart the values of the number of defects per unit against the unit sequence (or time). In general, control charts for qualitative characteristics have asymmetric (non-symmetric) limits around the target line, because if the lower limit is equal to a negative value, it turns to zero.

3. The Application:

In this section, the common SQC covered in the theory side will be applied through two examples by using Minitab-17 package. These examples were taken from (Montgomery, 2013, P. 282 & P. 345).

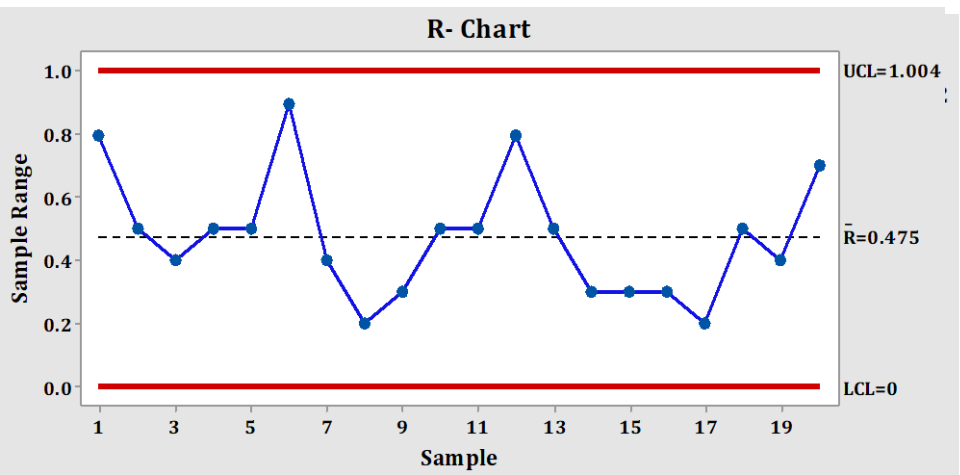
Example (1):

The net weight (in oz) of a dry bleach product to be monitored using a sample size (n=5). Data for (20) preliminary samples and some statistical measures are shown in the table (1).

Table (1): Data of 20 samples for dry bleach product (weight in oz)

Sample	x_1	x_2	x_3	x_4	x_5	$\bar{x}_j$	R_j	S_j
1	15.8	16.3	16.2	16.1	16.6	16.20	0.8	0.292
2	16.3	15.9	15.9	16.2	16.4	16.14	0.5	0.230
3	16.1	16.2	16.5	16.4	16.3	16.30	0.4	0.158
4	16.3	16.2	15.9	16.4	16.2	16.20	0.5	0.187
5	16.1	16.1	16.4	16.5	16.0	16.22	0.5	0.217
6	16.1	15.8	16.7	16.6	16.4	16.32	0.9	0.370
7	16.1	16.3	16.5	16.1	16.5	16.30	0.4	0.200
8	16.2	16.1	16.2	16.1	16.3	16.18	0.2	0.084
9	16.3	16.2	16.4	16.3	16.5	16.34	0.3	0.114
10	16.6	16.3	16.4	16.1	16.5	16.38	0.5	0.192
11	16.2	16.4	15.9	16.3	16.4	16.24	0.5	0.207
12	15.9	16.6	16.7	16.2	16.5	16.38	0.8	0.327
13	16.4	16.1	16.6	16.4	16.1	16.32	0.5	0.217
14	16.5	16.3	16.2	16.3	16.4	16.34	0.3	0.114
15	16.4	16.1	16.3	16.2	16.2	16.24	0.3	0.114
16	16.0	16.2	16.3	16.3	16.2	16.20	0.3	0.123
17	16.4	16.2	16.4	16.3	16.2	16.30	0.2	0.100
18	16.0	16.2	16.4	16.5	16.1	16.24	0.5	0.207
19	16.4	16.0	16.3	16.4	16.4	16.30	0.4	0.173
20	16.4	16.4	16.5	16.0	15.8	16.22	0.7	0.303

By applying the formulas of the central line, lower control limit (LCL) and upper control limit (UCL) for the variable charts, we get the following charts:



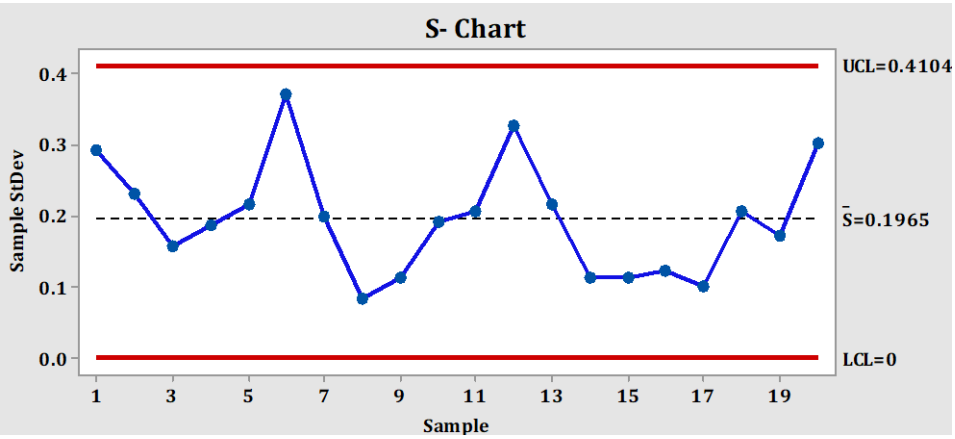


Figure (1): Statistical quality control charts of variables

Example (2):

The data in Table (2) represent the results of inspecting all units of a personal computer produced for the past ten days.

Table (2): Data for personal computer inspecting results

Day	Units inspected	Nonconforming units	Fraction Nonconforming
1	80	4	0.050
2	110	7	0.064
3	90	5	0.056
4	75	8	0.107
5	130	6	0.046
6	120	6	0.050
7	70	4	0.057
8	125	5	0.040
9	105	8	0.076
10	95	7	0.074

By applying the formulas of the central line, lower control limit (LCL) and upper control limit (UCL) for the attribute charts, we get the following charts:

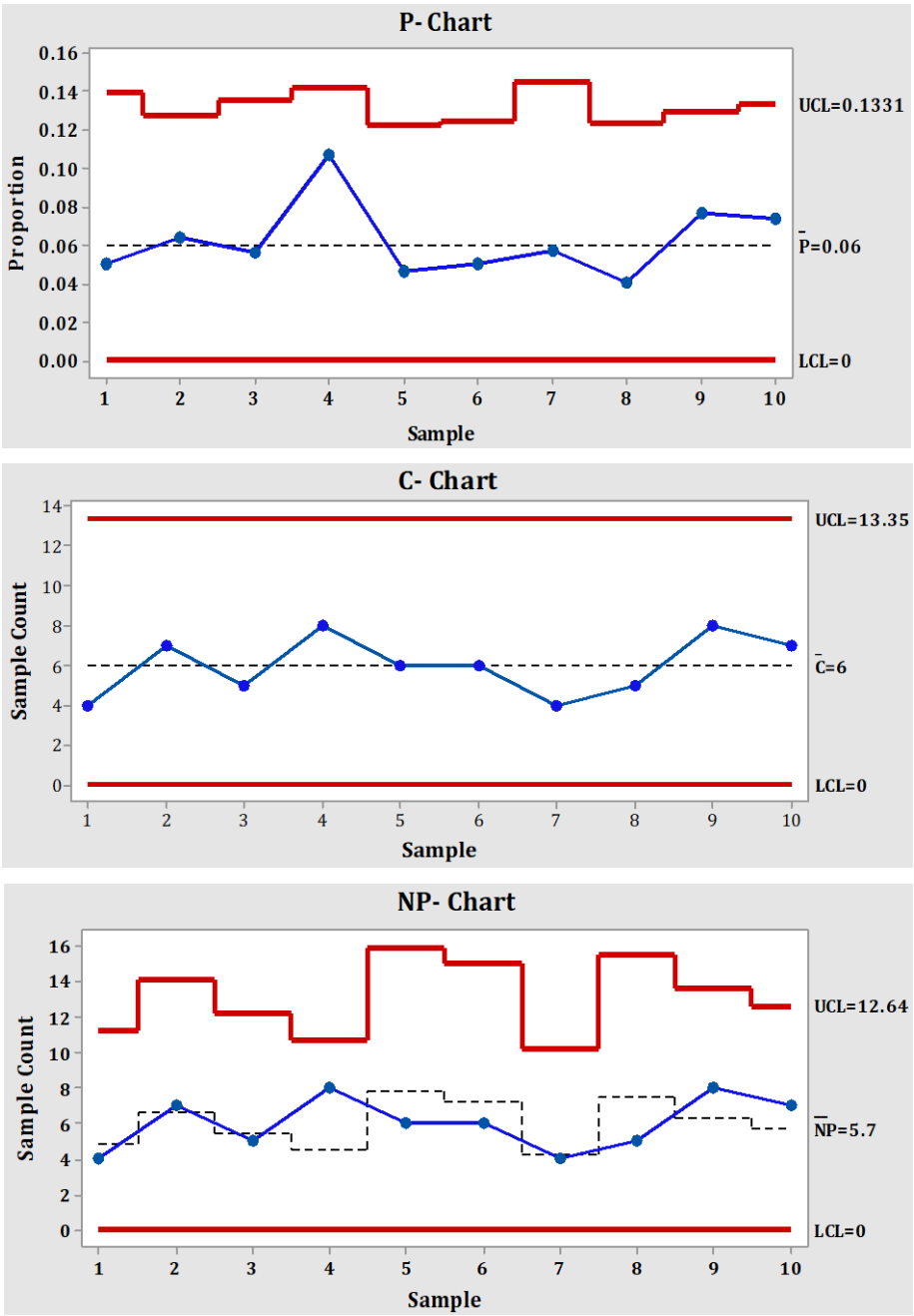


Figure (2): Statistical quality control charts of attributes

Conclusions:

Statistical quality control charts are a tool for learning about the behaviour of a measure over time. They allow us to differentiate between common cause and special cause variation, striking a balance between reacting too early to random fluctuations and reacting too late to real shifts in performance. In the context of quality improvement work, they allow us to tell if the changes we're testing produce variation in the desired direction – variation which cannot be explained by chance fluctuations in the data. Shewharts' charts make use of both the average level of performance and the amount of natural variation expected in the data. Because the amount of variation expected to arise naturally depends on the type of measure being graphed, we need to select the appropriate type of chart for the type of data being graphed. Statistical theory underlies the rules used to differentiate common cause variation from special cause variation. Special cause variation is always a learning opportunity. In the context of a quality improvement project, the goal is to identify changes to a process, which produce special cause variation in the desired direction.

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